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AN ANALYSIS OF THIRD ORDER INTERMODULATION IN VACUUM TUBES

A THESIS

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the Faculty of the Graduate Division

by

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AN ANALYSIS OF THIRD ORDER

INTERMODULATION IN VACUUM TUBES

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Date Approved by Chairman: May 29, 1958

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ABSTRACT

Analytic Techniques to evaluate tube parameters mathematically were investigated. A program that contains the various mathematical steps and that utilizes the IBM-650 magnetic-drum computer was developed. Data obtained on several tubes demonstrate that it is possible with these methods to obtain, as a function of bias, the third-order coefficient of the Taylor's series which represents the transfer characteristic. This parameter is important in the study of the intermodulation characteristics of amplifier tubes. In connection with the study of intermodulation characteristics of mixers, sufficient data was available from results to make a qualitative evaluation of the fourth-order term.

CHAPTER I

INTRODUCTION

In the last few years interference has become a very acute problem in radio communication. This interference is not basically a problem of overcrowding, since theoretically the available spectrum will accommodate many more voice communication channels than are presently used. The problem is that interference to communication in a channel may be caused by communication on different channels. Thus transmitters which cause less interference and receivers which are less susceptible to interference must be designed to remedy the problem.

Many types of interference have been noted. Most of this interference can be traced directly to nonlinearities in either the transmitter or receiver circuits. A few examples are (1) harmonics of the transmitter fundamental frequency which are produced in the power amplifier, (2) sideband "splatter" caused by the modulator, (3) cross-modulation produced in the radio frequency tubes of a receiver, and (4) intermodulation produced in radio frequency tubes and first mixer of receivers.

Transmitters which cause very little interference have been designed and built. Lowpass and bandpass filters which eliminate the harmonics from the power output of a transmitter can be built; since the harmonics must differ substantially in frequency from the fundamental. Unfortunately a substantial difference in frequency does not exist between the desired receiver frequency and the frequency which may cause interference. Both cross-modulation and intermodulation are caused by signals in the channels which are adjacent to the desired channel.

Cross-modulation is the variance of the transconductance of the first radio frequency tube at an audio rate by the modulation on an adjacent channel signal. This variance of the transconductance will cause the modulation of the undesired signal to become modulated on the desired signal. Intermodulation is the mixing of two adjacent channel signals in either the radio frequency tubes or mixer to given an on channel signal.

A study conducted at the Engineering Experiment Station of the Georgia Institute of Technology found third order intermodulation to be the most serious problem at the UHF frequencies.[1] This problem is so serious that a system designed for the UHF band failed to operate effectively with fewer than ten percent of the available channels in use. Operation with even one megacycle channel is seriously handicapped by interference.

Work in information theory and speech analysis has shown that a channel of less than 100 cycles can be used to transmit speech information. This means that the band from 225-400 mc could carry at least 1,750,000 simultaneous conversations where now it can carry less than 100. Improved modulation techniques will soon carry the problem of third-order intermodulation to lower and lower frequencies. It is doubtful that the state of the art of designing filters will catch up with that of designing new modulation schemes for many years if ever, so that the problem of intermodulation produced in the radio frequency and mixer tubes of a receiver must be solved if we are to possess the communication faculties that are and will be needed.

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[1] Mauldin, H. W., Jr. and Meek, R. E., Study Program for Investigation to Aid in Reduction and Prevention of UHF Interference. Technical Report No. 2, Project No. A-109, Engineering Experiment Station, Georgia Institute of Technology, Atlanta, Georgia, 1955.

CHAPTER II

ANALYSIS OF AMPLIFIERS AND MIXERS WITH APPLIED SIGNALS

It has long been known that vacuum tubes are nonlinear devices. It is the nonlinear characteristic of a tube which gives rise to harmonic production and frequency translation of signals by mixing. Many present day circuits depend on this nonlinearity. A few examples are mixers in superheterodyne receivers, harmonic frequency multipliers, and some forms of modulators. In these cases it is advantageous to accentuate certain orders of the nonlinearities present in vacuum tubes, for example, by selecting an optimum operating point. For amplification of a signal, however, it is desired that the tube be a linear device. Of course this cannot be realized and certain spurious responses such as third-order intermodulation may seriously degrade the usefulness of the tube as an amplifier. In locations or in systems where third-order intermodulation is the most serious form of interference it is advisable to bias the tubes so that the third-order curvature will be a minimum. To do this it is useful to know the third-order curvature as a function of bias. Before proceeding to this, the causes of intermodulation in amplifiers and mixers will be examined. A typical plate current characteristic is shown in Figure 1. A power series representation of the

$$i_b = \sum_{n=0}^{k} C_n e_c^{\,n} \quad , \qquad (1)$$

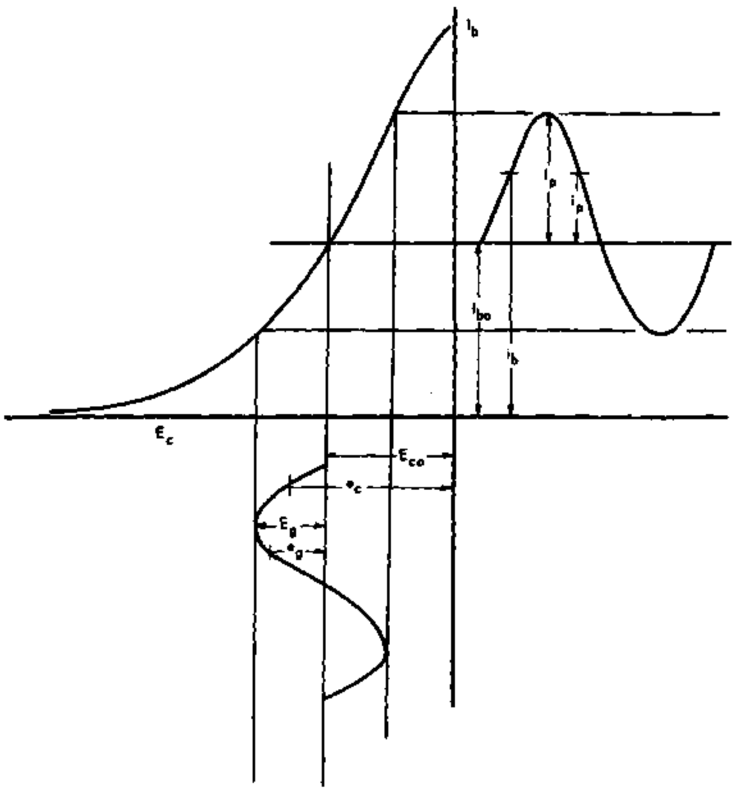

Figure 1. Fundamental Components in a Tube Characteristic.

which is a Maclaurin's series. The plate current characteristic may also be expressed by

$$i_b = \sum_{n=0}^{k} C_n(e_c - E_{co})^n \ . \tag{2}$$

This is a Taylor's series representation of the transfer curve expanded about E_{co}. For each E_{co} there will be a different set of C_n's, since

$$C_n = \frac{1}{n!} \left. \frac{\partial^n i_b}{\partial e_c{}^n} \right|_{e_c = E_{co}} \ . \tag{3}$$

To determine the expressions for third-order, fifth-order, and seventh-order intermodulation for an amplifier in terms of the C_n's a signal of the form

$$e_g = E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + E_3 \sin \omega_3 t + E_4 \sin \omega_4 t \tag{4}$$

may be assumed at the input to tube. The total grid voltage, e_c, then is

$$e_c = e_g + E_{co} = E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + E_3 \sin \omega_3 t + E_4 \sin \omega_4 t + E_{co} \ . \tag{5}$$

Substitution of Equation 5 in Equation 2 gives

$$i_b = \sum_{n=1}^{k} C_n(E_1 \sin \omega_1 t + E_2 \sin \omega_2 t + E_3 \sin \omega_3 t + E_4 \sin \omega_4 t)^n . \tag{6}$$

Expression 6 may be expanded and the terms of like frequency can be collected in the form

$$i_b = \sum_{x=-k}^{k} \sum_{y=-l}^{l} B_{(x,y)} \sin(x\omega_1 t + y\omega_2 t) , \tag{7}$$

where B is a function of E_1, E_2, E_3, E_4 and C_n. From Equation 7 the third-, fifth- and seventh-order intermodulation-frequency currents are

$$\begin{aligned} &B_{(2,-1)} \sin(2\omega_1 t - \omega_2 t), \\ &B_{(3,-2)} \sin(3\omega_2 t - 2\omega_3 t), \text{ and} \\ &B_{(4,-3)} \sin(4\omega_3 t - 3\omega_4 t). \end{aligned} \tag{8}$$

The object now is to determine the values of the $B_{(x,y)}$'s. This can be done by selecting a value of k + l in Equation 7 and actually performing the indicated expansion. Expansion of Equation 7 for k and l = 12 has been performed, and the values obtained for the third-, fifth, and seventh-order coefficients are:

$3^{\underline{rd}}$ order IM due to curvature through $11^{\underline{th}}$ order = $B_{(2,-1)}$ =

$$C_3\ 3/4\ E_1^{\ 2}E_2 + C_5(5/4\ E_1^{\ 4}E_2 + 15/8\ E_1^{\ 2}E_2^{\ 3}) + C_7(105/64\ E_1^{\ 6}E_2 + 105/32\ E_1^{\ 2}E_2^{\ 5} + 105/16\ E_1^{\ 4}E_2^{\ 3}) + C_9(63/32\ E_1^{\ 8}E_2 \tag{9}$$

$$+ 1280/256\ E_1{}^2E_2{}^7 + 3780/256\ E_1{}^6E_2{}^3 + 504/32\ E_1{}^4E_2{}^5)$$

$$+ C_{11}(1155/512\ E_1{}^{10}E_2 + 12705/1024\ E_1{}^2E_2{}^9$$

$$+ 3465/128\ E_1{}^8E_2{}^3 + 11550/512\ E_1{}^4E_2{}^7$$

$$+ 34650/512\ E_1{}^6E_2{}^5)\ ,$$

$5^{\underline{th}}$ order IM = $B_{(3,-2)}$ =

$$C_5\ 5/8\ E_2{}^3\ E_3{}^2 + C_7(165/64\ E_2{}^5E_3{}^2 + 35/16\ E_2{}^3E_3{}^4) \tag{10}$$

$$+ C_9(756/256\ E_2{}^7E_3{}^2 + 1266/256\ E_2{}^3E_3{}^6$$

$$+ 630/64\ E_2{}^5E_3{}^4) + C_{11}(1155/256\ E_2{}^9E_3{}^2$$

$$+ 1155/128\ E_2{}^3E_3{}^8 + 6930/256\ E_2{}^7E_3{}^4$$

$$+ 3460/1024\ E_2{}^5E_3{}^6)\ ,$$

$7^{\underline{th}}$ order IM = $B_{(4,-3)}$ =

$$C_7\ 35/64\ E_3{}^4E_4{}^3 + C_9(252/128\ E_3{}^6E_4{}^3 + 680/256\ E_3{}^4E_4{}^5) \tag{11}$$

$$+ C_{11}(1155/256\ E_3{}^8E_4{}^3 + 6930/1024\ E_3{}^4E_4{}^7$$

$$+ 6930/512\ E_3{}^6E_4{}^5),$$

where

$$E_1 = \text{interfering signal of frequency } \omega_1 = \omega_o + \Delta\omega\ , \tag{12}$$

$$E_2 = \text{interfering signal of frequency } \omega_2 = \omega_o + 2\Delta\omega\ ,$$

E_3 = interfering signal of frequency $\omega_3 = \omega_o + 3\Delta\omega$,

E_4 = interfering signal of frequency $\omega_4 = \omega_o + 4\Delta\omega$,

$\Delta\omega$ = channel spacing, and

ω_o = desired signal frequency.

The responses of $B_{(2,-1)}$, $B_{(3,-2)}$, and $B_{(4,-3)}$ can be easily shown to be of angular frequency ω_o by substitution of the above frequencies.

$B_{(2,-1)}$ is of angular frequency (13)

$$(2\omega_1 - \omega_2) = 2(\omega_o + \Delta\omega) - (\omega_o + 2\Delta\omega) = \omega_o$$

$B_{(3,-2)}$ is of angular frequency

$$(3\omega_2 - 2\omega_3) = 3(\omega_o + 2\Delta\omega) - 2(\omega_o + 3\Delta\omega) = \omega_o$$

$B_{(4,-3)}$ is of angular frequency

$$(4\omega_3 - 3\omega_4) = 4(\omega_o = 3\Delta\omega) - 3(\omega_o + 4\Delta\omega) = \omega_o$$

This shows that intermodulation creates a response which is of the same frequency as the desired signal and this once created can not be filtered out.

Experiment has shown that when the interfering signal levels are small a good approximation to the intermodulation distortion is

$$B_{(2,-1)} = 3^{\underline{rd}} \text{ Order} = 3/4 \; E_1{}^2 E_2 C_3 \; , \tag{14}$$

$$B_{(3,-2)} = 5^{\underline{th}} \text{ Order} = 5/8 \; E_2 E_3{}^2 C_5, \text{ and} \tag{15}$$

$$B_{(4,-3)} = 7^{\underline{th}} \text{ Order} = 35/64 \; E_3{}^4 E_4{}^3 C_7 \; . \tag{16}$$

However, C_3, C_5, and C_7 can be expressed as a function of the bias voltage alone by means of Equation 3. Hence to find the optimum bias for minimum intermodulation of the third-, fifth-, and seventh-order it is only necessary to find the proper roots of

$$C_3 = f(E_{co}) = 0 \ , \tag{17}$$

$$C_5 = f(E_{co}) = 0, \text{ and} \tag{18}$$

$$C_7 = f(E_{co}) = 0. \tag{19}$$

Experimental results have shown that the most serious type of intermodulation is third-order intermodulation. These results have also indicated that C_5, C_7, - - - C_{2n-1} can be neglected with respect to C_3, so that the third-order intermodulation term can be approximated as $3/4\ C_3 E_1{}^2 E_2$.

The mathematical analysis of intermodulation in mixers proceeds in a manner similar to that used above. For mixers the input to the grid is of the form

$$e_c = E_{co} + E_1 \sin(\omega_o + \Delta\omega)t + E_2 \sin(\omega_o + 2\Delta\omega)t \tag{20}$$

$$+ E_{Lo} \sin(\omega_{Lo})t \ ,$$

where ω_{Lo} is the local oscillator frequency. Substituting Equation 20 into Equation 2 gives

$$i_b = \sum_{n=0}^{k} C_n[E_1 \sin(\omega_o + \Delta\omega)t\ E_2 \sin(\omega_o + 2\Delta\omega)t \tag{21}$$

$$+ E_{Lo} \sin \omega_{Lo} t]^n \ .$$

Expanding Equation 21 and collecting terms of like frequency results in

$$i_b = \sum_{m=-k}^{k} \sum_{n=-1}^{1} \sum_{p=-m}^{m} B_{(m,n,p)} \sin[(m + n)\omega_o \tag{22}$$

$$+ (m + 2n)\ \omega + p\omega_{Lo}]t$$

If the tube is being used as a mixer, the frequency component of Equation 22 that is of interest is the i-f frequency. Hence the term of interest is of the form

$$B_{(m,n,p)} \ \mathrm{Cos}(\omega_o - \omega_{Lo})t, \text{ or:} \tag{23}$$

$$B_{(m,n,p)} \ \mathrm{Cos}(\omega_{Lo} - \omega_o)t \ .$$

However, the magnitude of the $B_{(m,n,p)}$ coefficient of either of these terms is the same, and hence a minimum of one coincides with a minimum of the other. Comparing Equation 22 with the term

$$B_{(m,n,p)} \ \mathrm{Cos}(\omega_o - \omega_{Lo})t,$$

it is seen that the two are equal when

$$m + n = 1, \tag{24}$$

$$m + 2n = 0, \text{ and}$$

$$p = -1.$$

Solution of Equation 24 gives

$$m = 2 \tag{25}$$

$n = -1$, and

$p = -1$.

Consequently, the coefficient of interest in determining the magnitude of the fourth-order intermodulation product is $B_{(2,-1,-1)}$. If the indicated expansion of Equation 21 is actually performed for a given value of $\underline{k}$, $\underline{l}$, and $\underline{m}$ it is found that $B_{(2,-1,-1)}$ is a function of C_4, C_6, C_8 and higher even-order coefficients and also of the magnitudes E_1, E_2, and E_{Lo}. Fortunately, $B_{(2,-1,-1)}$ is given to a good approximation by

$$B_{(2,-1,-1)} = \frac{3E_1{}^2E_2E_{Lo}C_4}{2} . \tag{26}$$

But from Equation 3 it is seen that C_4 is a function of E_{co} alone. Hence, the minimum value of $D_{(2,-1,-1)}$ with respect to the grid bias occurs for that value of E_{co} which is the proper root of

$$C_4 = f(E_{co}) = 0. \tag{27}$$

For determination of intermodulation rejection in amplifiers C_3 must be known, and for mixers C_4 must be known. If C_3 or C_4 have zeros then the optimum bias for amplifier or mixer operation respectively will be at these zeros.

CHAPTER III

METHODS OF DETERMINING C_3 AND C_4

With the use of very carefully instrumented systems, it was found that the third-order intermodulation of some vacuum tubes varied as much as 60 db as the grid bias was varied. Figure 2 shows the test setup used to obtain third, fifth, and seventh-order intermodulation as a function of bias. Extreme care was taken to prevent intermodulation from taking place anywhere but in the vacuum tube. Typical results of these measurements are shown in Figures 3 and 4. A similar result was obtained for fourth-order intermodulation as can be seen from Figure 5.

This study is mainly concerned with the development of a technique by which the third-order intermodulation characteristic can be obtained from the transfer curve (I_b versus E_c) of a vacuum tube. It is hoped that such a method will make time consuming measurements of the third-order intermodulation characteristics of vacuum tubes unnecessary, yield a better insight into the fundamental nature of the tube systems, and provide a useful tool in the development of low intermodulation tubes and other special purpose vacuum tubes.

Several well known methods, when used as described in the literature, gave erratic and erroneous results. Two methods, however, were devised by which the C_3 characteristics could be determined. One of the methods provides information concerning the zero of C_4.

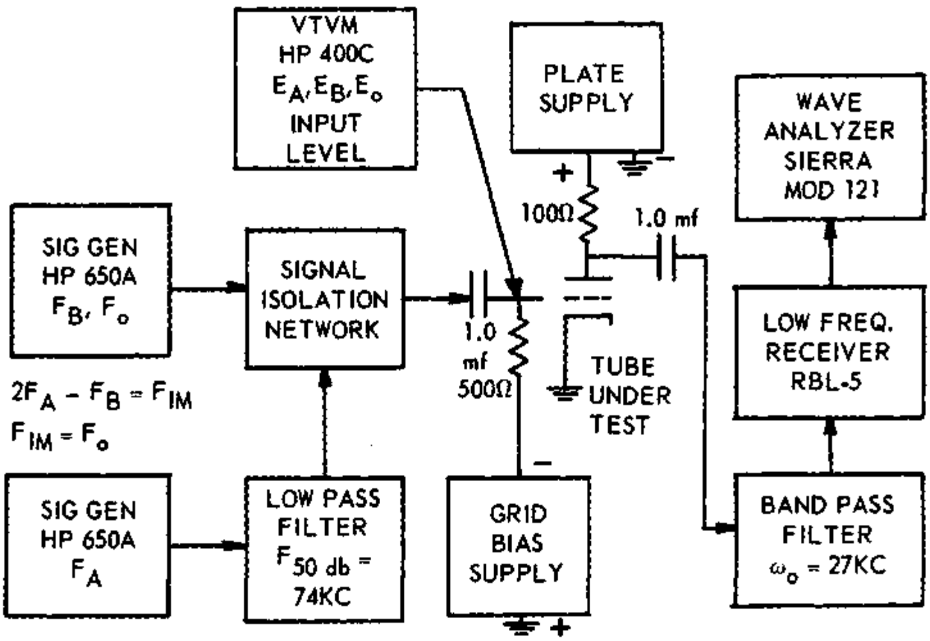

Figure 2. Instrument Arrangement Diagram for Low Frequency Intermodulation Rejection Measurements.

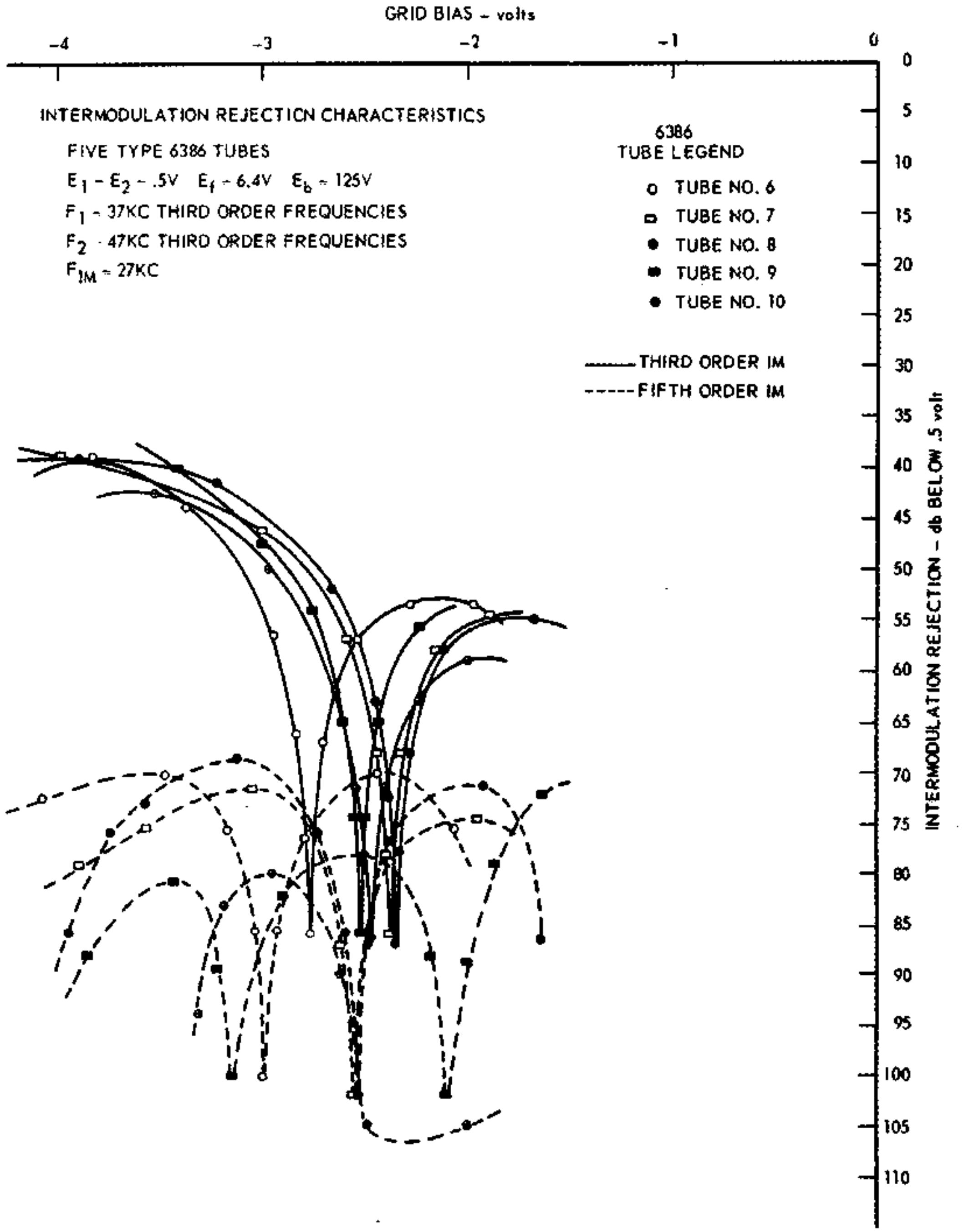

Figure 3. 6386 Low Frequency Third-Order Intermodulation Characteristics.

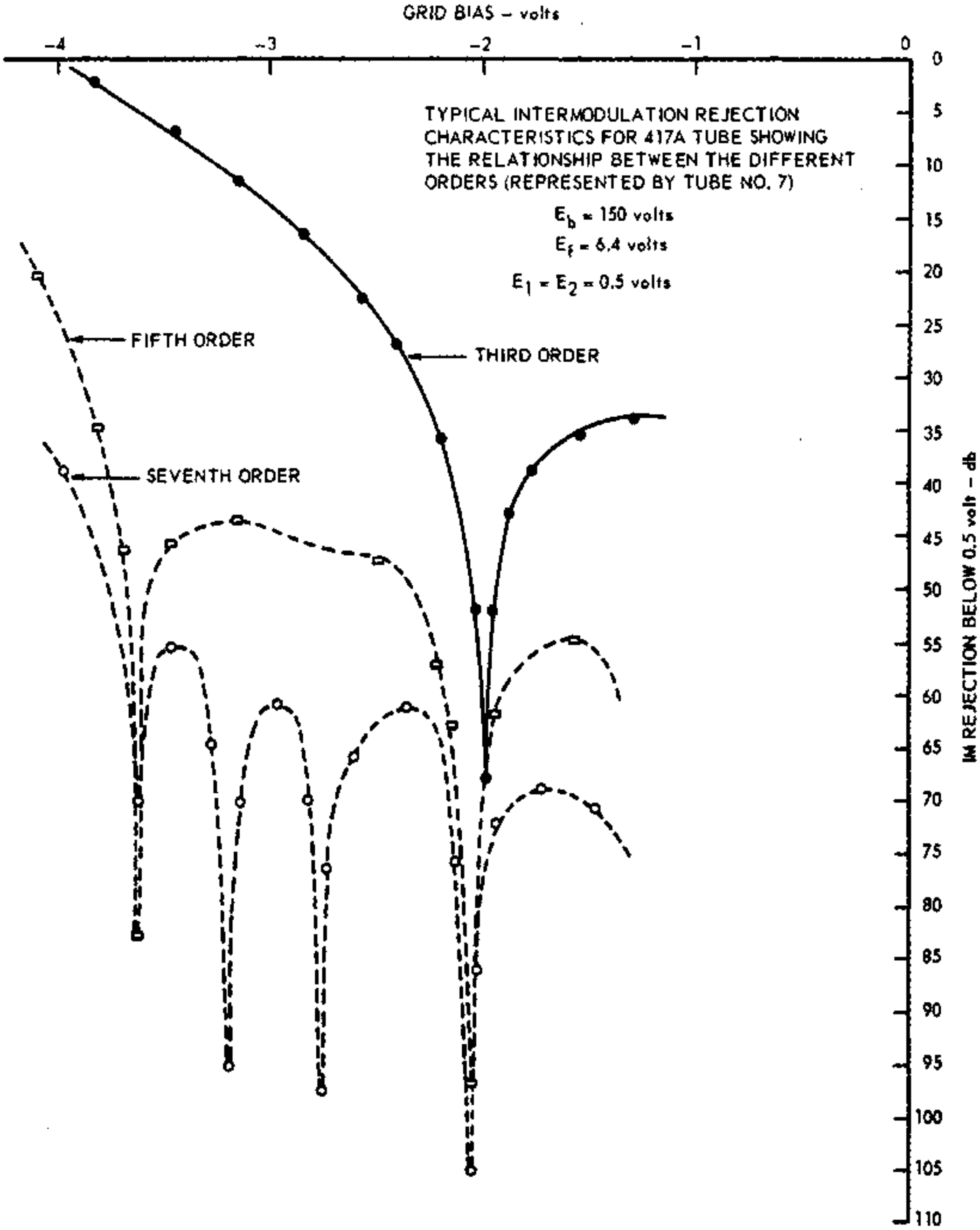

Figure 4. Typical Intermodulation Rejection Characteristics for 417A Triode.

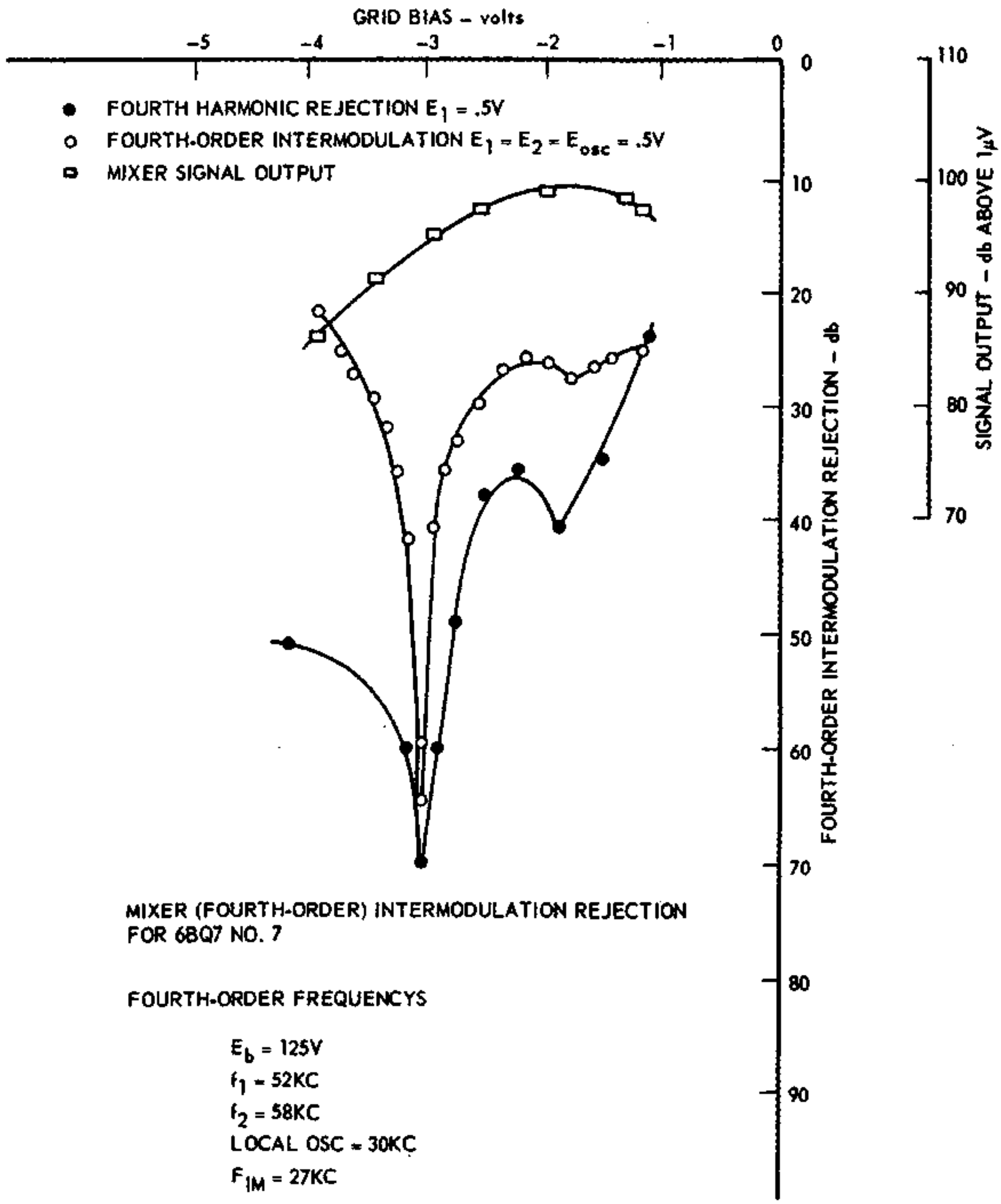

Figure 5. Mixer Fourth-Order Intermodulation Rejection for 6BQ7.

Using well developed methods of tube instrumentation and advanced ideas of numerical analysis coupled with modern high speed computers, a program to find the third-order intermodulation characteristics as a function of bias was constructed. Examples of the results of applying these techniques to particular tube types are shown and inherent limitations arising are given.

Early efforts to obtain the third-order curvature of a vacuum tube used static characteristics which were measured point by point. It was realized that some drift and reading errors existed in this type of data, and an attempt was made to smooth the experimental curves by generating a least-squares polynomial approximation with an IBM 650 Digital Computer. Some smoothing was accomplished, but when the least-squares curve was differentiated (to obtain the Taylor series coefficients) the results were very erratic and misleading. A search was then made for a better experimental method of obtaining a transfer curve and for a better method of deriving a polynomial approximation to fit the transfer curve.

A method of obtaining a transfer curve was developed which made use of an analog computer and its associated equipment. The grid voltage of the tube being studied was swept from cutoff to zero bias by a voltage that increased linearly with respect to time. The output current of the tube was plotted versus time, and conversion of the time scale to bias voltage gave the desired transfer curve, which was found to be very smooth. Three runs of the transfer curve were made with each of several tubes, and the repeated curves of each set exactly overlapped.

To preserve the advantage of the smooth curves obtained on the analog computer, it was necessary to read values at the various points very closely. This was accomplished by using a low-power calibrated microscope. The reading error was estimated to be about plus or minus four percent of the smallest scale division of the graph paper, or about the width of the plotted line.

The previously mentioned unfavorable results obtained with methods appearing in the literature can be partially attributed to small unavoidable reading errors in the data. Most of these methods use difference equations and are thus very susceptible to small errors in the data. It was assumed that these methods might yield reliable results if applied to a smoothed data curve then approximated the transfer characteristic. This led to an investigation of methods for approximating a curve whose value was known only at a finite number of points.

Three possible types of approximations are (1) exponential series, (2) transcendental series, and (3) power series.

The shape of the transfer curve indicates that it might be approximated by an exponential series. Unfortunately no satisfactory method of an exponential approximation of a curve was found in the literature.

A Fourier series approximation of the curve was attempted. The basis of this method is the simulation of a transfer curve by the leading edge of a Pseudo square wave. The axis was shifted to obtain even symmetry.[2] A ten-term approximation to the transfer curve by this method was not a close enough fit to be used in obtaining the C_n's. Since little improvement could be obtained with the addition of a few more terms, more favorable forms of approximation were sought.

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[2] Barrow, W. L., "Contribution to the Theory of Nonlinear Circuits with Large Applied Voltages," Proceedings of the Institute of Radio Engineers, 22, (1934), 964-980.

A review of the literature on approximation functions showed that many methods of power series approximation existed. One of the best known of these, the least-squares method, was tried. A program written for the IBM 650 in the PALS language system existed in the library of the Rich Electronic Computer Center at Georgia Tech.[3] This program is capable of finding an nth degree least-squares polynomial from any M observations provided $n < M \leq 26$. The residues between the data and the polynomial approximation at the input points are also provided. These residues were found to be very small. However, when the computed fifth-, seventh-, and ninth-degree polynomials were each differentiated three times to obtain equivalent power series coefficients (C_1, C_2, C_3), no correlation existed between the derivatives. Furthermore, none of these gave results which agree with the experimental third-order intermodulation tests or analog computer data. Trials with other polynomial approximating routines also showed that even though very good approximating curves could be obtained it was not possible to differentiate the polynomial expressions analytically and obtain reliable results. The basic reason for this failure was the approximation was not of the same order as the true curve. It is believed that the transfer curve cannot be expressed as a finite power series.

A very simple approximation which yields results which have a similar appearance to experimental results has been advanced recently

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[3] VonHoldt, R. E., and Brousseau, R. J., "Weighted Least-Square Polynomial Approximation to a Continuous Function of a Single Variable," UCRL-4711, (1956).

by A. G. Anisimov.[4] This method seeks to approximate the transfer curve by

$$I_b - I_0[1 \tanh(\frac{S_o}{I_o}) E_c] \tag{28}$$

where S_o and I_o are the transconductance and plate current at Zero bias and taking the ratio of the first and third derivatives we have

$$\frac{\frac{dI_b}{dE_c}}{\frac{d^3I_b}{dE_c}} = 2(\frac{S_o}{I_o})^2 [3 \tanh^2(\frac{S_o}{I_o} E_c) - 1] \tag{29}$$

This curve obviously has a zero at

$$\tanh^2 \frac{S_o E_c}{I_o} = \frac{1}{3} \tag{30}$$

Results with the transfer curves of triodes have shown this approximation to be only a fair approximation of the bias point at which the zero of the third order curvature occurs. This approximation has been presented to show what is known about recent work taking place in Russia to alleviate third-order intermodulation.

The library of the Rich Electronic Computer Center contained a routine for approximating a curve with a fifth-degree polynomial. This

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[4] Anisimov, A. G., "Certain Problems of Tuned Amplification," Radio Engineering, 12, 54-48, August 1957.

program (written in the FACS language[5] for the IBM 650) gave a much closer fit to the transfer curve than did the least-squares method, and when used in conjunction with Espley's method gave results which agreed fairly well with experimental data. The limitation of this program to a fifth-degree polynomial was felt to be a drawback, since there was no way of knowing if higher order approximations of this type would give still better results.

A set of data points, suitably normalized, was programmed for obtaining a Tshebysheff-Polynomial[6] approximation to a curve. Routines for finding fifth-, seventh-, and ninth-degree fits were formulated in the Bell General Purpose Language.[7] These routines calculated the approximating polynomial, evaluated the polynomial for a given increment, and yielded the residues at the input points. The residues that were obtained were very small; as can be seen from the approximation of 6AF4A characteristic shown in Table I, the largest residue within the range of interest was approximately equal to the estimated error in reading the transfer curve. The Espley method,[8] when applied to these

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[5] Boask, Robert, "Development of a Floating Decimal Abstract Coding System (FACS)," Technical News Letter No. 10, (1955), 28.

[6] Kennedy, E. C., Tajen, Joseph, and Barnes, A. M., "11-, 15-, and 19-Point Gramm-Tshebysheff Polynomial Worksheet," Unpublished material, three tables, Ordnance Aerophysics Laboratory, (1952).

[7] Hamming, R. W. and Wiess, R. A., "The General Purpose System for the 650," Bell Telephone Laboratories, (Unpublished Report).

[8] Espley, D. C., "The Calculation of Harmonic Production in Thermionic Valves with Resistive Loads," Proceedings of the Institute of Radio Engineers, 21, (1933), 1439-1446.

Table 1. Approximation of 6AF4A Characteristics with Tshebysheff-Polynomial

Bias Point Volts	Input Data (ma)	Tshebysheff-Polynomial Evaluated at Input Points (ma)	Residues (ma)
1.0	62.92900	62.924259	.005259
2.0	54.2875	54.268750	-.018755
3.0	46.0120	46.021664	-.006640
4.0	38.2610	38.289373	.028370
5.0	31.2800	31.266070	-.013930
6.0	25.1350	25.105109	-.029891
7.0	19.8885	19.866541	-.021959
8.0	15.46750	15.519455	.051955
9.0	11.92650	11.977694	.051194
10.0	9.18400	9.147530	-.036470
11.0	7.1375	6.965874	-.2171626
12.0	5.1425	5.407388	.264880
13.0	4.5950	4.439660	-.155304
14.0	3.8645	*	*
15.0	3.3055	*	*

* Not evaluated by program

Tshebysheff-Polynomials, showed a seventh-degree fit of this type to be preferable to the fifth-degree fit or to the fifth-degree system fit. Indeed, the seventh-degree Tshebysheff-Polynomial fit was found to be better than the ninth-degree Tshebysheff-Polynomial fit even though the residues for the ninth-order polynomial were less than those of the seventh.

The reason conjectured for this difference was that the slope of the curve as well as its magnitude is an important element. The constraint of the ninth-order curve to a close fit at a discrete number of points caused the slope of the ninth-degree approximation curve in relation to the true curve to be in greater error than the slope of the seventh-degree approximation. It is assumed that both the slope and the magnitude of the seventh-order curve were closer to true curve than they were in the fifth-degree polynomial.

The original tables of argument for the seventh-degree Tshebysheff-Polynomial are included in the Appendix as Table 6 and the programs used for these are contained in one master program for obtaining C_n's, are shown in the Appendix as Table 7.

As mentioned earlier, it was felt that if a suitable approximation to the transfer curve could be obtained, one or more of the methods in the literature might give reliable results when applied to the approximation. A method proposed by Espley was investigated. The Espley method considers the operation of a tube at a bias, E_{co}, with a small signal, e_g, applied. Thus operation is over only a small part of the transfer curve. In Figure 6 the signal amplitude and the portion of the transfer curve over which operation takes place is greatly accentuated.

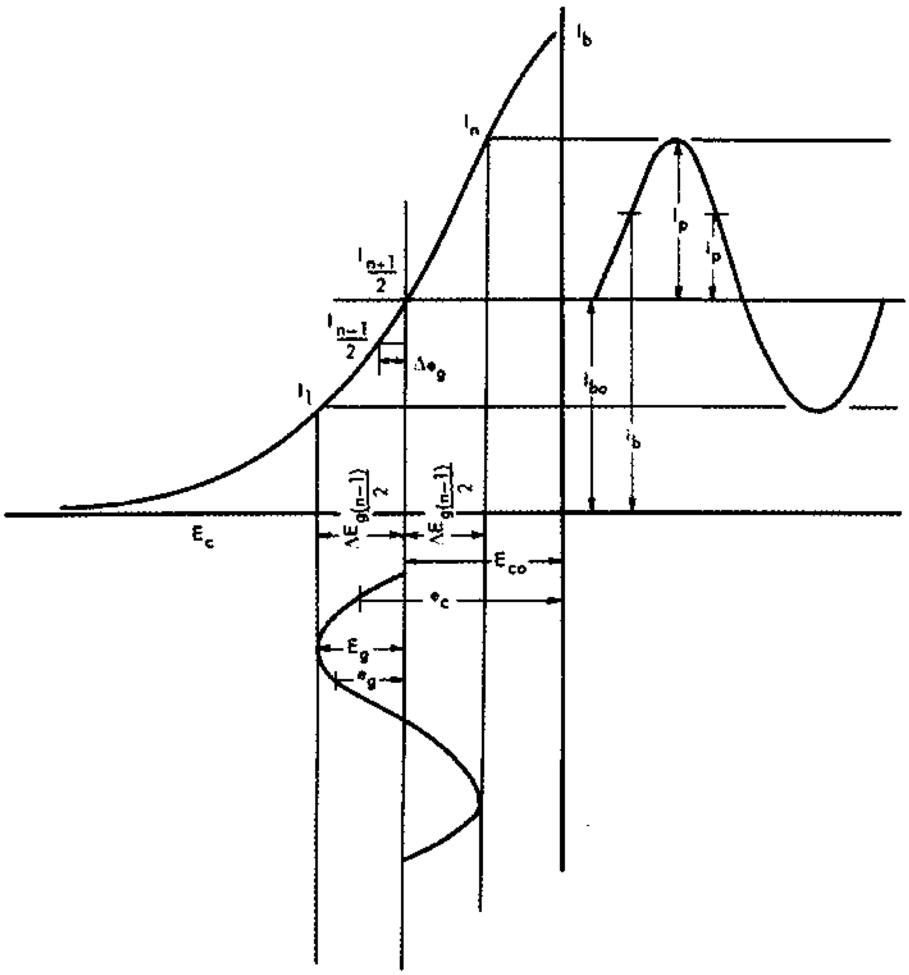

Figure 6. Fundamental Tube Characteristic Components Used in Espley's Relations.

The current at E_{co} is I_{bo} which is designated $I_{\frac{N+1}{2}}$, with the currents I_1, and I_N, being the minimum and maximum currents respectively in each cycle. N is an odd number. The signal voltage is divided into N-1 increments, Δe_g, and the output current is measured at $E_{co} \pm K\Delta e_g$ where K ranges from 0 to $\frac{N-1}{2}$. It is assumed that the operation of the tube about E_{co} can be approximated by an nth order Taylor's series expanded about E_{co}. Thus a set of equations for I_1, I_2——I_N takes the form

$$I_1 = C_o + C_1[+ \Delta e_g(\tfrac{N-1}{2})] + C_2[+ \Delta e_g(\tfrac{N-1}{2})]^2 + \ldots \qquad (31)$$

$$+ C_N[+ \Delta e_g(\tfrac{N-1}{2})]^N$$

$$I_2 = C_o + C_1[+ \Delta e_g(\tfrac{N-3}{2})] + C_2[+ \Delta e_g(\tfrac{N-3}{2})]^2 + \ldots$$

$$+ C_N[+ \Delta e_g(\tfrac{N-3}{2})]^N$$

.

.

$$I_{\frac{N+1}{2}} = C_o + C_1[0] + \ldots C_N[0]$$

.

. .

$$I_N = C_o + C_1[- \Delta e_g(\tfrac{N-1}{2})] + C_2[- \Delta e_g(\tfrac{N-1}{2})]^2 + \ldots$$

$$+ C_N[- \Delta e_g(\tfrac{N-1}{2})]^N.$$

This set of equations can be solved for the C_n's in terms of the currents. For N = 11 the formulas are shown below:

$$C_0 = I_6,$$

$$C_1 = \frac{1}{725760\ \Delta e_g}\Big\{576(I_{11} - I_1) - 7200(I_{10} - I_2 + 43200(I_9 - I_3) \quad (32)$$

$$- 172800(I_8 - I_4) + 604800(I_7 - I_5)\Big\} \quad ,$$

$$C_2 = \frac{1}{3628800(\Delta e_g)^2}\Big\{576(I_{11} + I_1) - 9000(I_{10} + I_2) + 72000(I_9 + I_3)$$

$$- 432000(I_8 + I_4) + 3024000(I_7 + I_5) - 5311152\ I_6\Big\} ,$$

$$C_3 = \frac{1}{725760(\Delta e_g)^3}\Big\{- 820(I_{11} - I_1) + 10088(I_{10} - I_2)$$

$$- 58428(I_9 - I_3) + 209712(I_8 - I_4) - 289392(I_7 - I_5)\Big\} ,$$

$$C_4 = \frac{1}{3628800(\Delta e_g)^4}\Big\{- 820(I_{11} + I_1) + 12610(I_{10} + I_2)$$

$$- 97380(I_9 + I_3) + 524280(I_8 + I_4) - 1401960(I_7 + I_5)$$

$$+ 1926540\ I_6\Big\} ,$$

$$C_5 = \frac{1}{34560(\Delta e_g)^5}\Big\{13(I_{11} - I_1) - 152(I_{10} - I_2) + 783(I_9 - I_3)$$

$$- 1872(I_8 - I_4) + 1938(I_7 - I_5)\Big\} ,$$

$$C_6 = \frac{1}{3628800(\Delta e_g)^6}\Big\{273(I_{11} + I_1) - 3990(I_{10} + I_2)$$

$$+ 27405(I_9 + I_3) - 98280(I_8 + I_4) + 203490(I_7 + I_5)$$

$$- 257796\ I_6\Big\} ,$$

$$C_7 = \frac{1}{120960(\Delta e_g)^7}\Big\{-5(I_{11} - I_1) + 52(I_{10} - I_2) - 207(I_9 - I_3)$$
$$+ 408(I_8 - I_4) - 378(I_7 - I_5)\Big\} ,$$

$$C_8 = \frac{1}{3628800(\Delta e_g)^8}\Big\{-30(I_{11} + I_1) + 390(I_{10} + I_2)$$
$$- 2070(I_9 + I_3) + 6120(I_8 + I_4) - 11340(I_7 + I_5)$$
$$+ 13860\ I_6\Big\} ,$$

$$C_9 = \frac{1}{725760(\Delta e_g)^9}\Big\{(I_{11} - I_1) - 8(I_{10} - I_2) + 27(I_9 - I_3)$$
$$- 48(I_8 - I_4) + 42(I_7 - I_5)\Big\} , \text{ and}$$

$$C_{10} = \frac{1}{3628800(\Delta e_g)^{10}}\Big\{1(I_{11} + I_1) - 10(I_{10} + I_2) + 45(I_9 + I_3)$$
$$- 120(I_8 + I_4) + 210(I_7 + I_5) - 252\ I_6\Big\} .$$

A program was written in the Bell General Purpose Language, which yielded reliable values for C_1's, C_2's, and C_3's corresponding to forty different bias points. Data needed for the program consisted of fifty evaluations of the seventh-degree Tshebysheff Polynomial approximation described above. This program is shown in the Appendix as Table 7.

Another method for obtaining the C's which made use of the analog computer program for finding the transfer curve was developed. The program for finding the transfer curve actually found $I_b(t)$ which could be converted to $I_b(E_c)$, but obviously if $I_b(t)$ was differentiated the derivative with respect to bias could be obtained by conversion of the time scale to bias.

Preliminary efforts showed that an accurate third derivative is unattainable with the available analog computer due to noise. The differentiator is inherently a noisy computer component and must be altered to obtain favorable results. A differentiator was devised using a second order loop connected in such a way that it is analogous to a perfect differentiator followed by a low pass LC filter. This arrangement removed the higher frequencies of noise and produced very little phase shift. It was found that still more noise could be removed by differentiating twice and integrating once for each derivative. This is the same in effect as using two LC low pass filters behind each perfect differentiator.

The program for finding the transfer curve C_1 and C_2 is shown in Figures 7 and 8. Runs were made very slowly so that phase errors would be minimized. Even though C_3 can not be found using available equipment, it is obvious that the zero of C_3 will occur at the maximum of C_2.

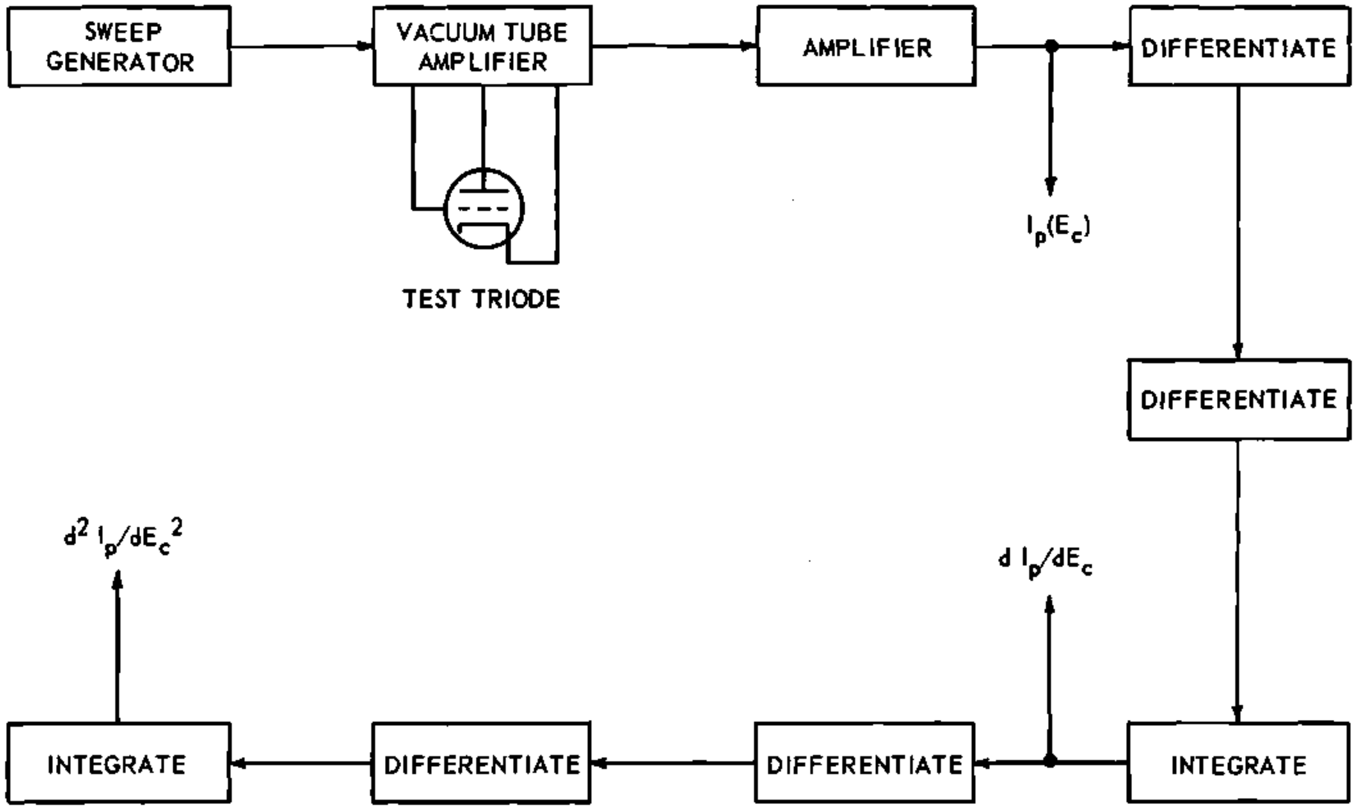

Figure 7. Analog Computer Flow Diagram for Tube Evaluation.

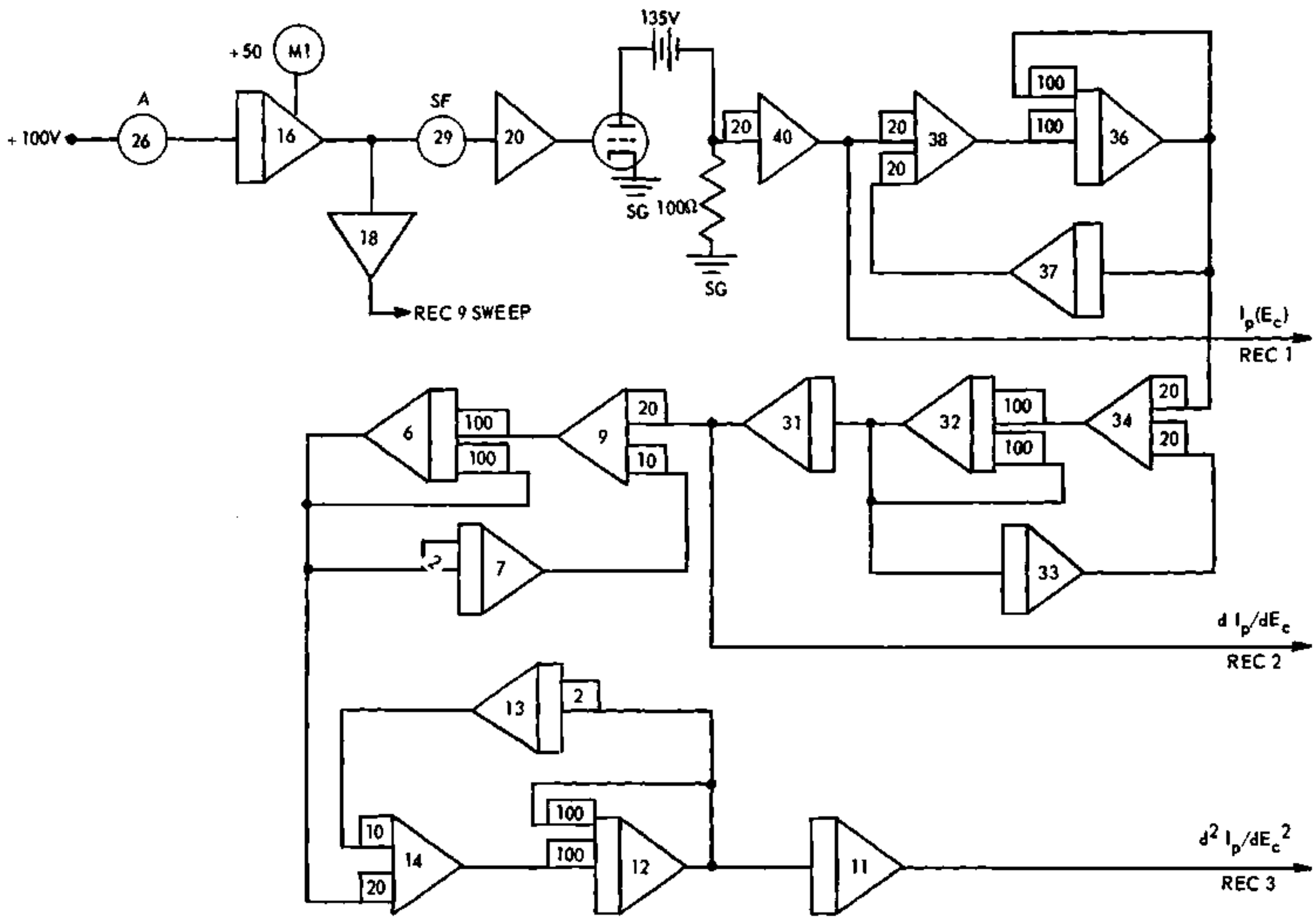

Figure 8. Functional Diagram of Analog Computer System for Investigation of Tube Characteristic.

CHAPTER IV

RESULTS

The 6AF4, 6AJ4, 6AN4, 6BC4, 6BQ7A, 6BY4, 6J4, 417A, and the 6386 vacuum tubes were selected as typical VHF and UHF tubes. The transfer curve, along with its first and second derivatives, for a typical tube of each type was run on the analog computer. Then the tube was tested to determine its third-order intermodulation characteristic and a selected group was tested to determine the fourth-order intermodulation characteristics. The third-order characteristic is associated with third-order intermodulation in amplifiers. The fourth-order characteristic is associated with third-order intermodulation in mixers.

The transfer curve data were read and put into a program which (1) found a seventh-degree Tshebysheff-Polynomial approximation to the curve, (2) evaluated the polynomial at fifty values of bias, (3) computed the residues at the fifteen input data points, and (4) then computed the desired C_n(C_1 or C_2 or C_3) at forty bias values. Approximately three minutes is required to run this program after the Bell General Purpose System is read into the computer. The computer will automatically repeat the program when a new set of data is available at the input device. A set of typical results for the third-order intermodulation characteristic is shown in Figure 9. This figure shows the transfer curve for a 6AF4A tube, its second derivative as obtained on the analog computer, C_2 as obtained mathematically, C_3 also as obtained

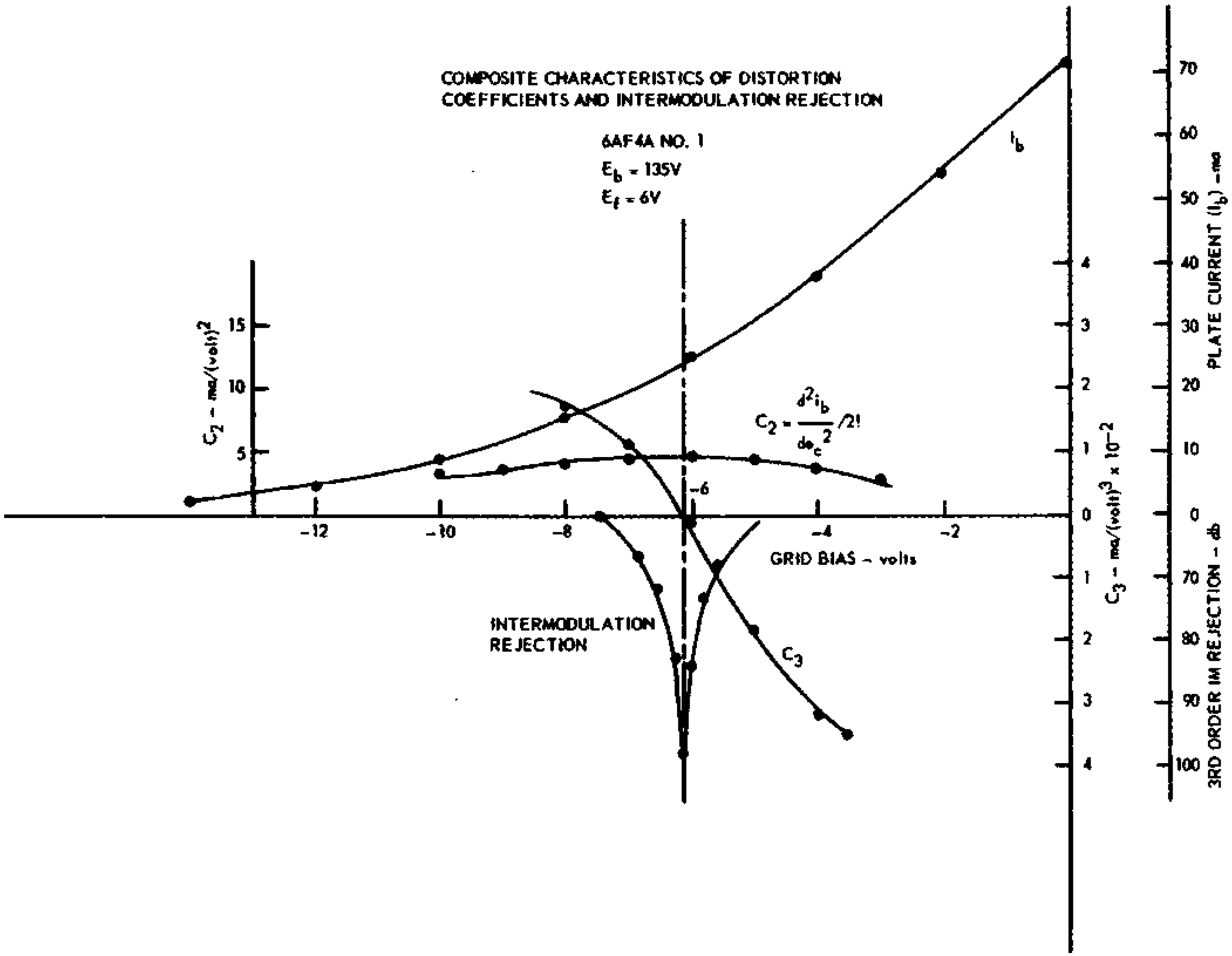

Figure 9. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6AF4A.

mathematically, and the third-order intermodulation characteristic as obtained from independent experimental tests on the tubes. C_2 was found to be exactly equal to 1/2 the second derivative, as predicted from the theory. The maximum third-order intermodulation rejection was found to occur at exactly the zero of C_3 and at the maximum of C_2, also as predicted from theory. This shows that the bias for maximum third-order intermodulation rejection can be found from either C_2 or C_3. Several tubes (6AJ4, 6AN4, 6BC4, 6BQ7A, 6J4, 417A, and 6386) with equally good correlations and a few with slightly poorer results are shown in Appendix as Figures 12 through 18. Table 2 also shows the correlation obtained between experimental, mathematical, and analog computer data.

C_4 was not directly obtainable by this method, but even as the maximum of C_2 shows the zero of C_3, so the maximum of C_3 shows the zero of C_4. Both fourth-order intermodulation and fourth harmonic rejection tests were run on some of the tubes. The bias voltages for maximum fourth-order intermodulation and fourth harmonic rejection were compared with the bias voltage for the maximum of C_3, and the correlation is apparent in the results shown in Table 3.

Experimental results show that the bias for maximum third-order intermodulation rejection is independent of the signal level for small signal operation of the tube. Results for several tubes are shown in Figure 10. A similar result was found by changing the evaluation increment of the Tshebysheff and thus changing the swing or psuedo signal level of the Espley method. These results are shown in Table 4. Thus precise signal level control is not needed for optimum operation of the receiver

Table 2. Comparison of Methods for Finding Optimum Bias Point for Third-Order Intermodulation Rejection

	Measured Third-Order Rejection	Measured with Analog Computer Maximum of C_2	Calculated $C_3 = 0$	Calculated Maximum of C_2
6AF4A	6.05	6.03	6.03	6.03
6AJ4	2.1	2.1	1.85	2.1
6AN4	1.30	1.25	1.07	1.125
6BC4	2.05	1.90	2.03	2.05
6BQ7A	2.60	2.5	2.675	2.5
6BY4	.875	.90	1.02	.90
417A	1.625	1.65	1.85	1.85
6386	2.52	2.52	2.52	2.52

Table 3. Comparison of Bias Voltages for Minimum Fourth-Order Products

	DETERMINED EXPERIMENTALLY		DETERMINED MATHEMATICALLY
	Bias Voltages for Maximum Fourth Harmonic Rejection	Bias Voltages for Maximum Fourth-Order Intermodulation Rejection	Bias Voltages for Maximum C_3
417A No. 7	2.52	- -	2.40
6BC4 No. 8	2.71	- -	2.625
6BY4 No. 5	1.03	- -	1.40
6BQ7A No. 7	3.10	3.11	3.08
6AN4 No. 12	1.70	2.05	1.68
6AF4 No. 1	(+)	3.6	3.50
6386 No. 7 (++)	- -	- -	3.50
6386 No. 8	3.13	3.45	- -
6AJ4 No. 7	2.78, 1.65	(++)	1.50

(+) Fourth harmonic too low to be measured.

(++) Tube failure.

Table 4. Effect of Signal Level on Bias Point for Zero of C_3 for Signal of

	.25	.5	.625	.75	1.0	1.25	2.5
6AF4A			6.03			6.09	6.09
6AJ4	1.47	1.85			1.91		
6AN4	1.07	1.07		1.06	*		
6BC4	2.07	2.03		2.08	2.04		
6BQ7A	*	2.60		2.55	2.60		
6386	*	2.52		2.44			

* Zero not indicated due to evaluation increment

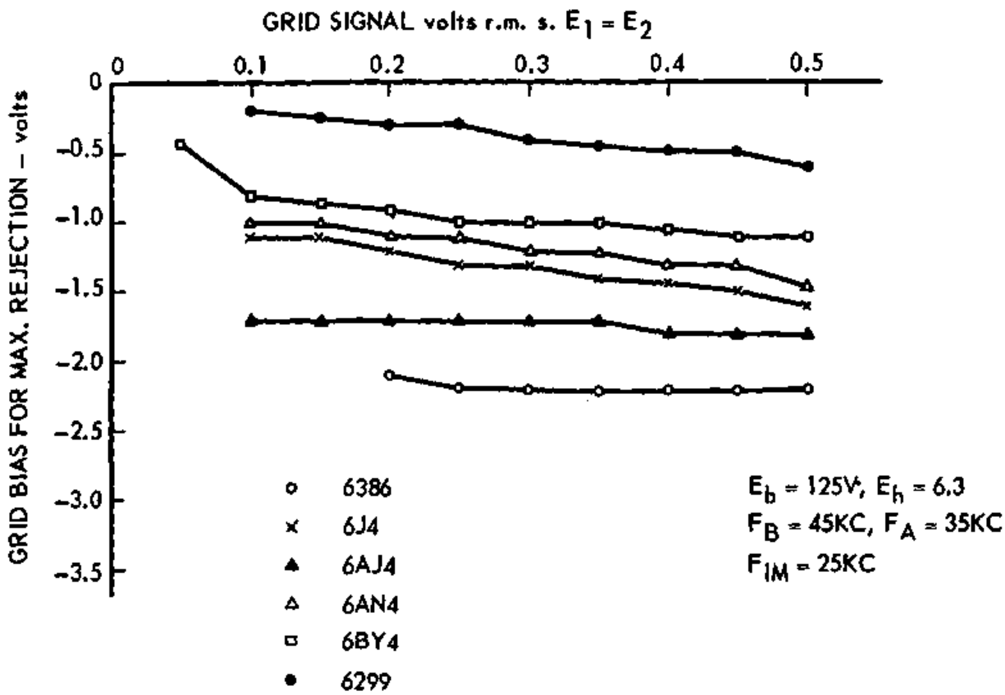

Figure 10. Grid Bias Versus Interfering Signal Level at Maximum Third-Order Intermodulation Rejection.

when signals which might cause third-order intermodulation interference are present.

One problem in high frequency receivers is high noise figures. This problem can be solved by the use of high amplification factor low noise tubes as the first radio frequency amplifier, because the noise figure of a receiver will approximately equal that of the first tube if sufficient gain is achieved in the first stage. Of course a very low noise figure or a high sensitivity is unimportant if interference drowns out signals which are many times stronger than the smallest signal the receiver will respond to if no other signals are present. Thus the choice of the first radio frequency tube must be a compromise between the tube which has the best third-order intermodulation rejection and the tube with the lowest noise and highest gain. In some of the tubes that were tested the valley of maximum third order intermodulation rejection was very narrow. This means that very critical biasing would be needed for optimum operation of these tubes. Thus a criterion for selection of a tube is taken as the width of the valley at the 60 db rejection points times the amplification factor of the tube. From Table 5 the 6AF4A can be seen to be far superior to other tubes types with respect to the above criteria. A very common first radio frequency tube, the 6J4, is the next best of the tubes tested. This indicates that if many present day receivers were operated with the bias fixed at the zero of C_3 of the first radio frequency tube a considerable amount of improvement would be gained in third-order intermodulation rejection of the receiver.

Table 5. Comparison of Amplification Factor, Width of Minimum Valley, and Maximum Third-Order Intermodulation Rejection

Tube Type	Amplification Factor (μ)	Width of Valley at 60 Db Points	(μ) Width of Valley	Maximum Third-Order Intermodulation
6AF4A	15	2.8	42	98
6AJ4	42	.125	5.26	81
6AN4	70	.025	1.75	75
6BC4	48	.025	1.2	67
6BQ7A	39	.035	1.52	69
6J4	55	.125	6.9	72
6386	17	.2	3.4	89

CONCLUSIONS AND RECOMMENDATIONS

Figure 11 shows the transfer curve and first four derivatives of three hypothetical tubes. It can be seen that slight changes in the shape of the transfer curve can drastically affect the values of C_1, C_2, C_3 and C_4. It can be seen that the second tube has by far the best third-order characteristic. It has a zero at low bias (high gain) and the values of C_3 near the zero are very small for a large range of voltage. Closer attention to the C_3 and/or C_4 characteristic in the design of new tubes might yield a very superior third-order capability.

An unexpected result of the comparison shown in Figure 11 is that since a tube with a linear characteristic cannot be built, one which appears to have an approximately linear characteristic (the first tube in Figure 11) may have worse intermodulation characteristics than a tube with a more parabolic shape (the second tube in Figure 11). This can be seen by comparing Tubes 1 and 2.

The methods described in this dissertation for finding C_3 might be very useful in their present form in the design of low intermodulation tubes. The mathematical method for finding C_3 could be refined by linking the analog computer, which would measure the tube's transfer curve, directly to the digital computer by an analog-to-digital converter. This would eliminate the time consuming process of reading data off the transfer by use of a microscope. An electronic third derivative might be possible using circuits and equipment especially designed for that

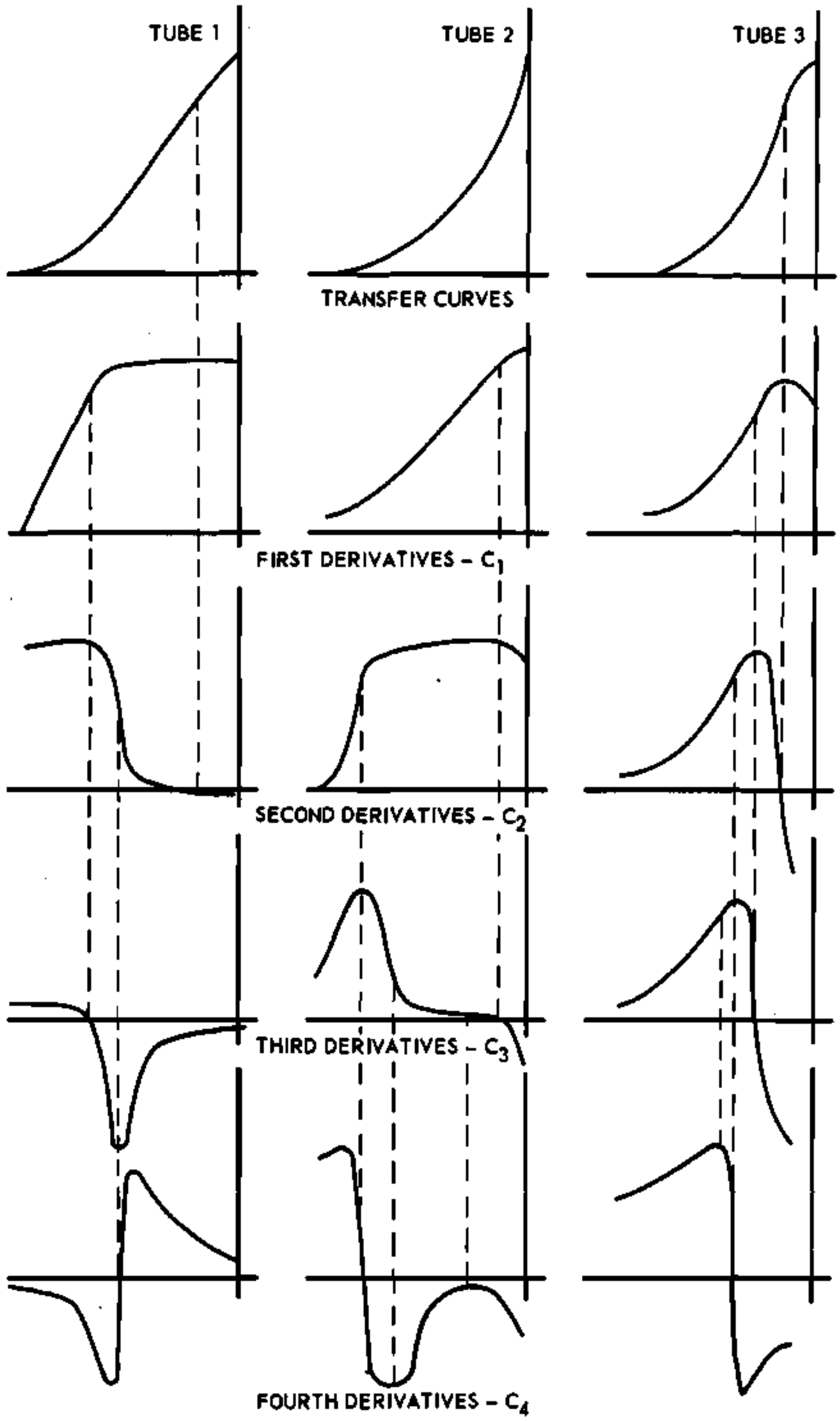

Figure 11. Hypothetical Tube Characteristics and Their Derivatives.

purpose. This equipment might be combined into an oscilloscope similar to the characteristic tracing oscilloscopes which are commercially available.

It is recommended that the radio frequency stages of all receivers whose performance is degraded by third-order intermodulation be operated with the radio frequency tubes and first mixer biased at the point of maximum third-order intermodulation rejection.

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APPENDIX

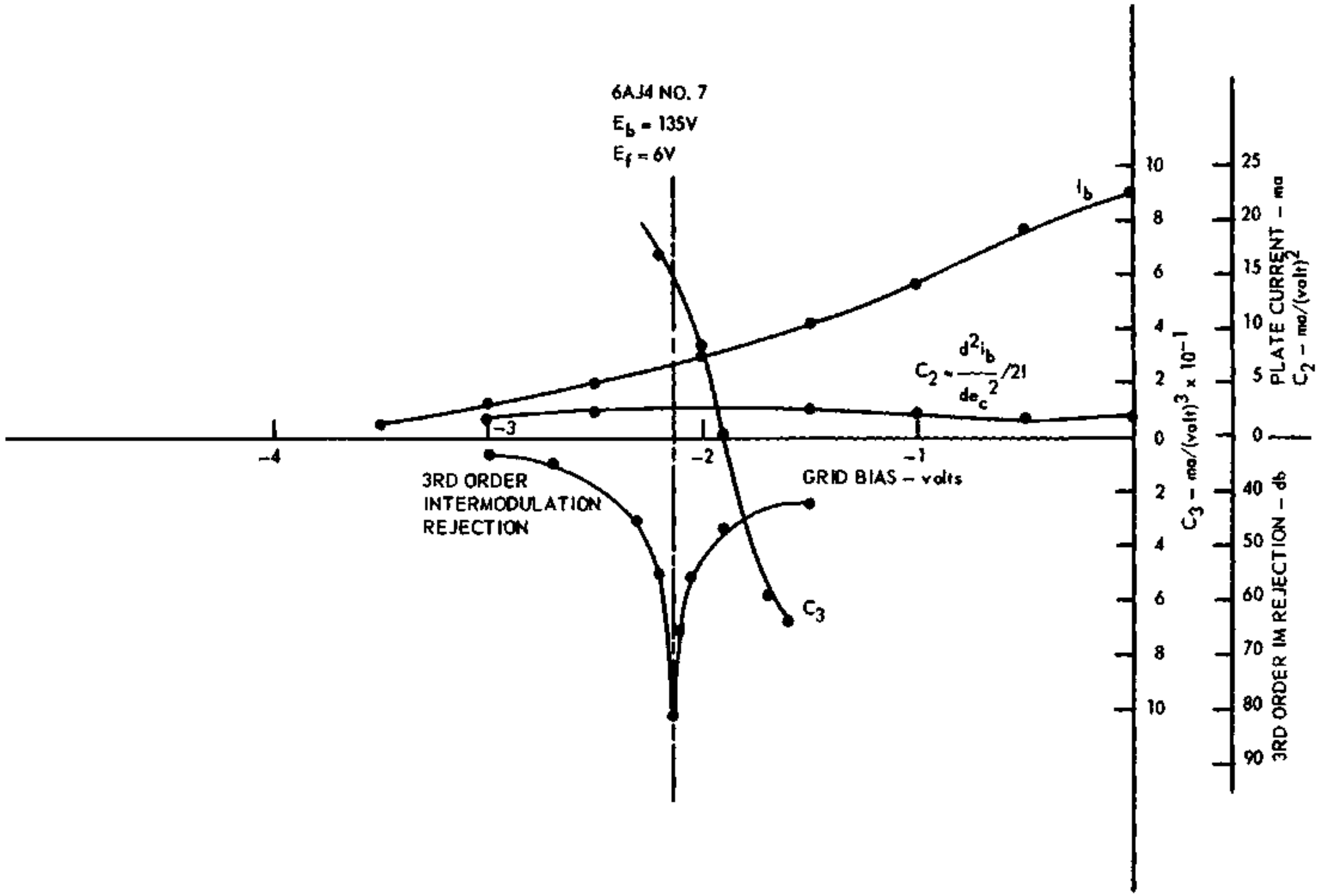

Figure 12. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6AJ4.

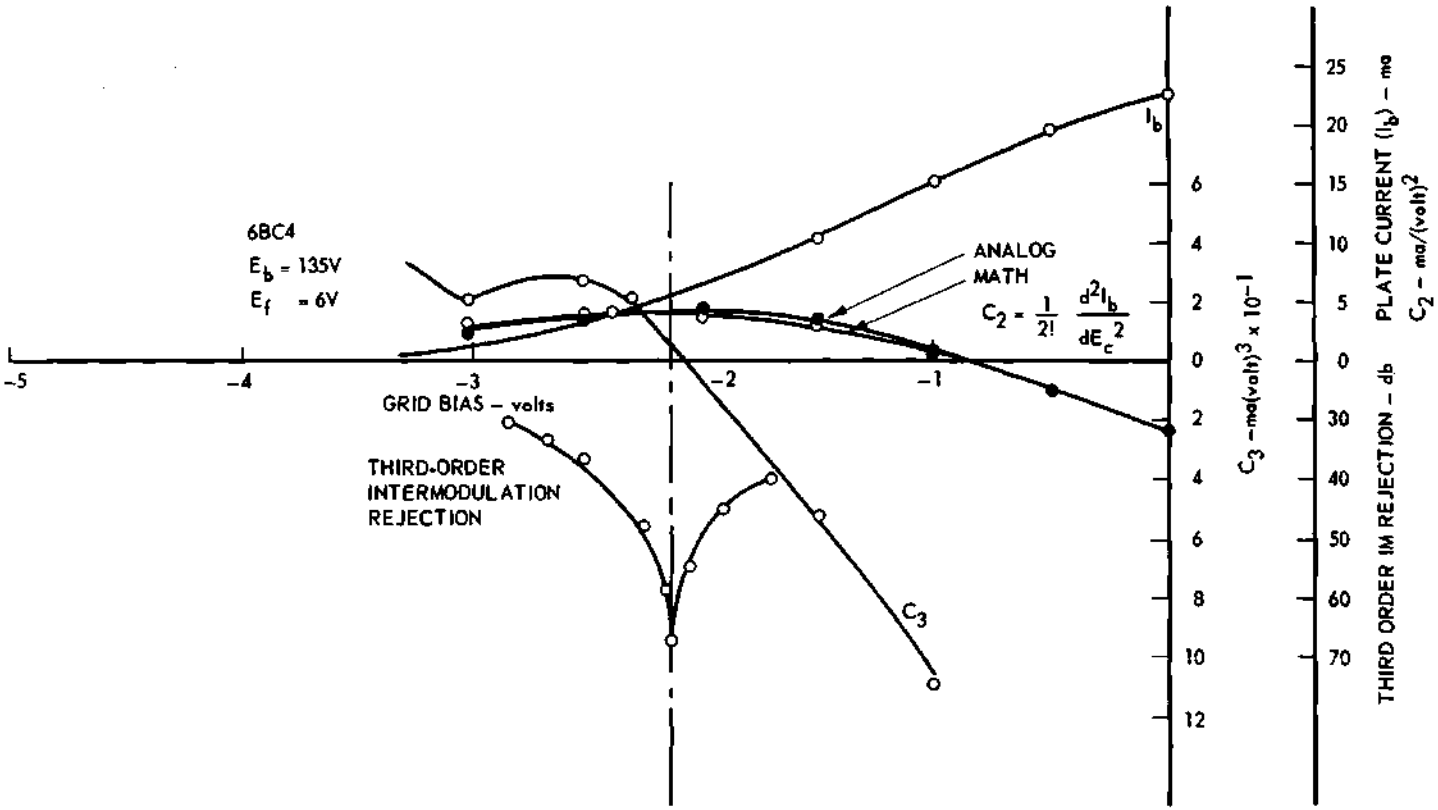

Figure 13. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6BC4.

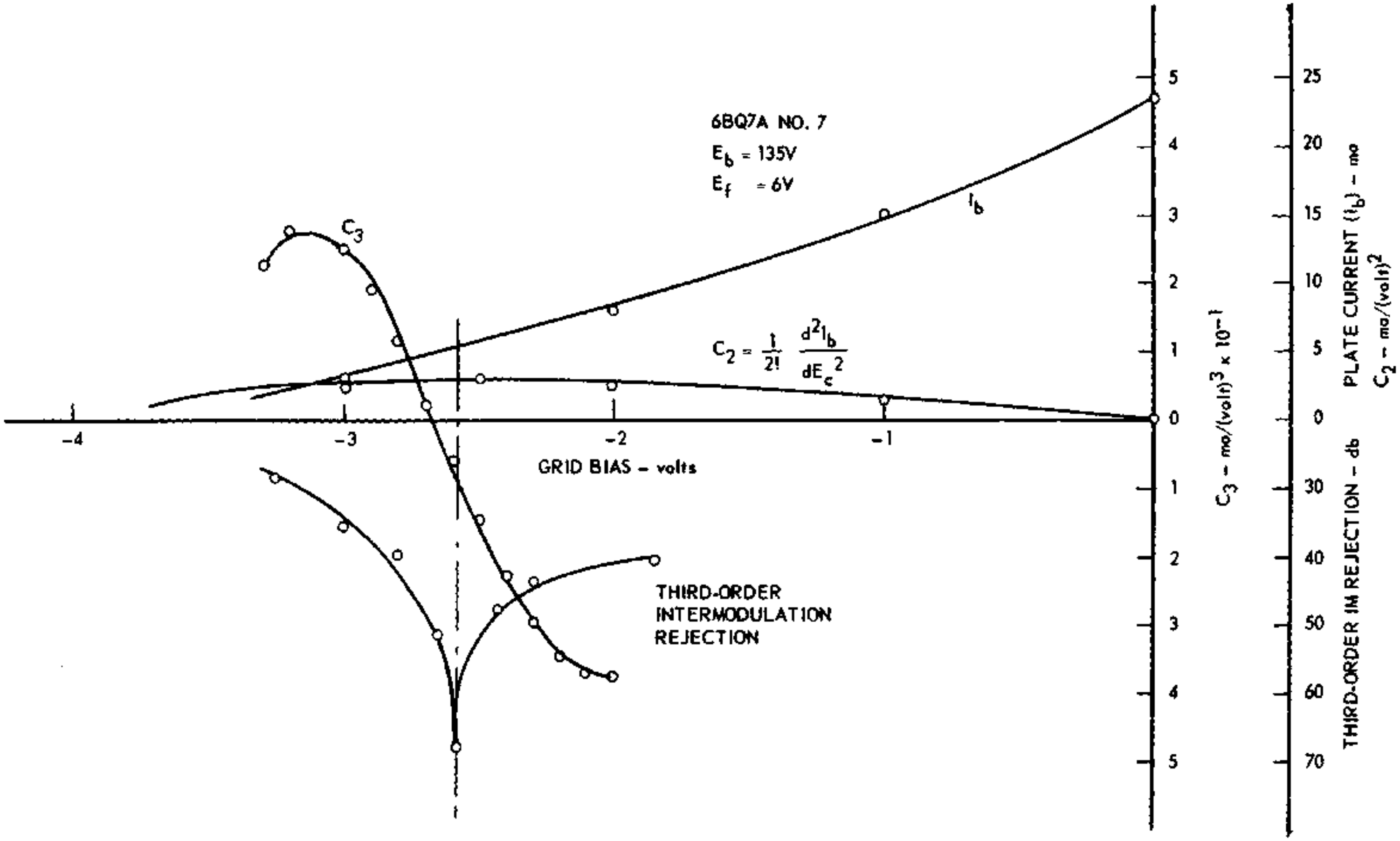

Figure 14. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6BQ7.

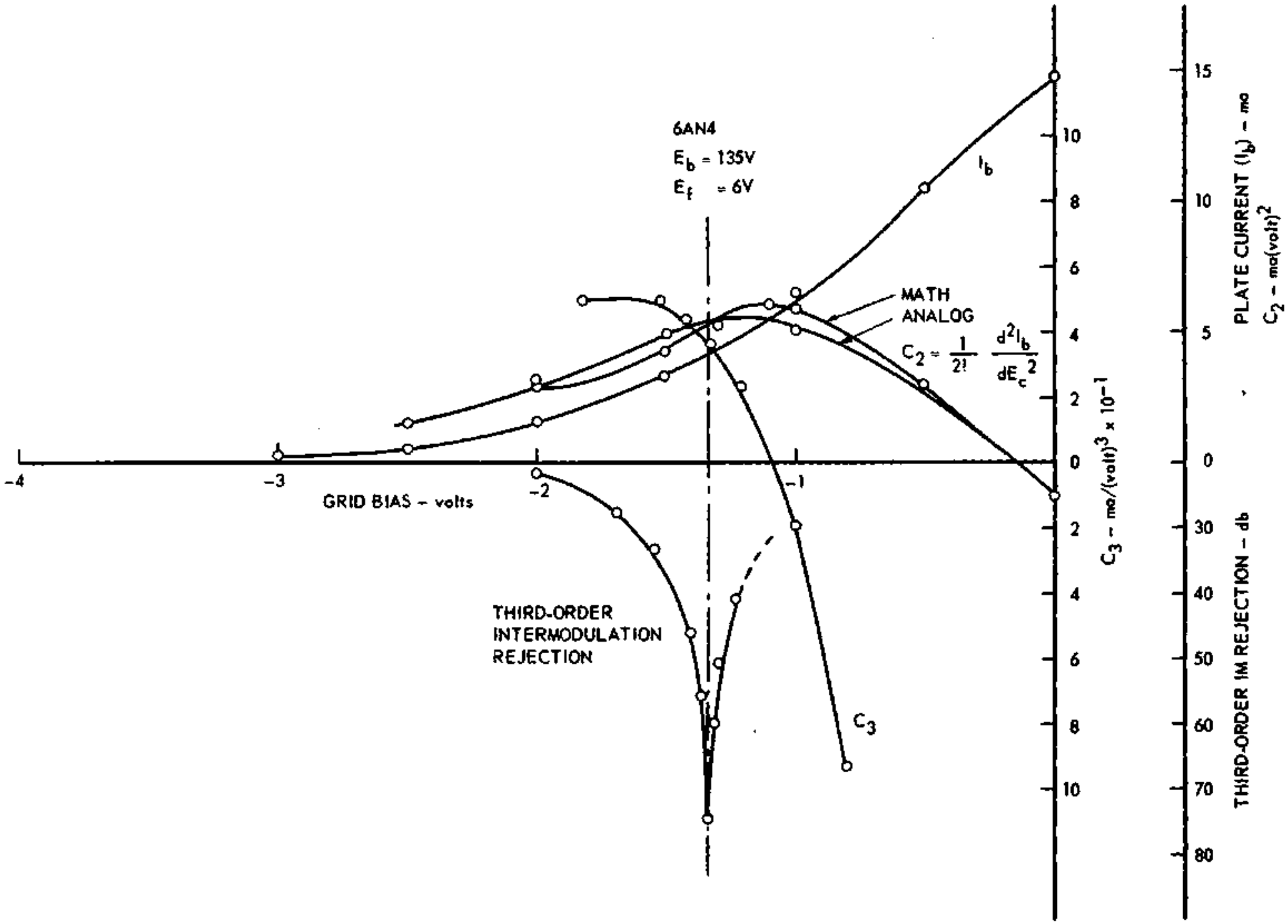

Figure 15. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6AN4.

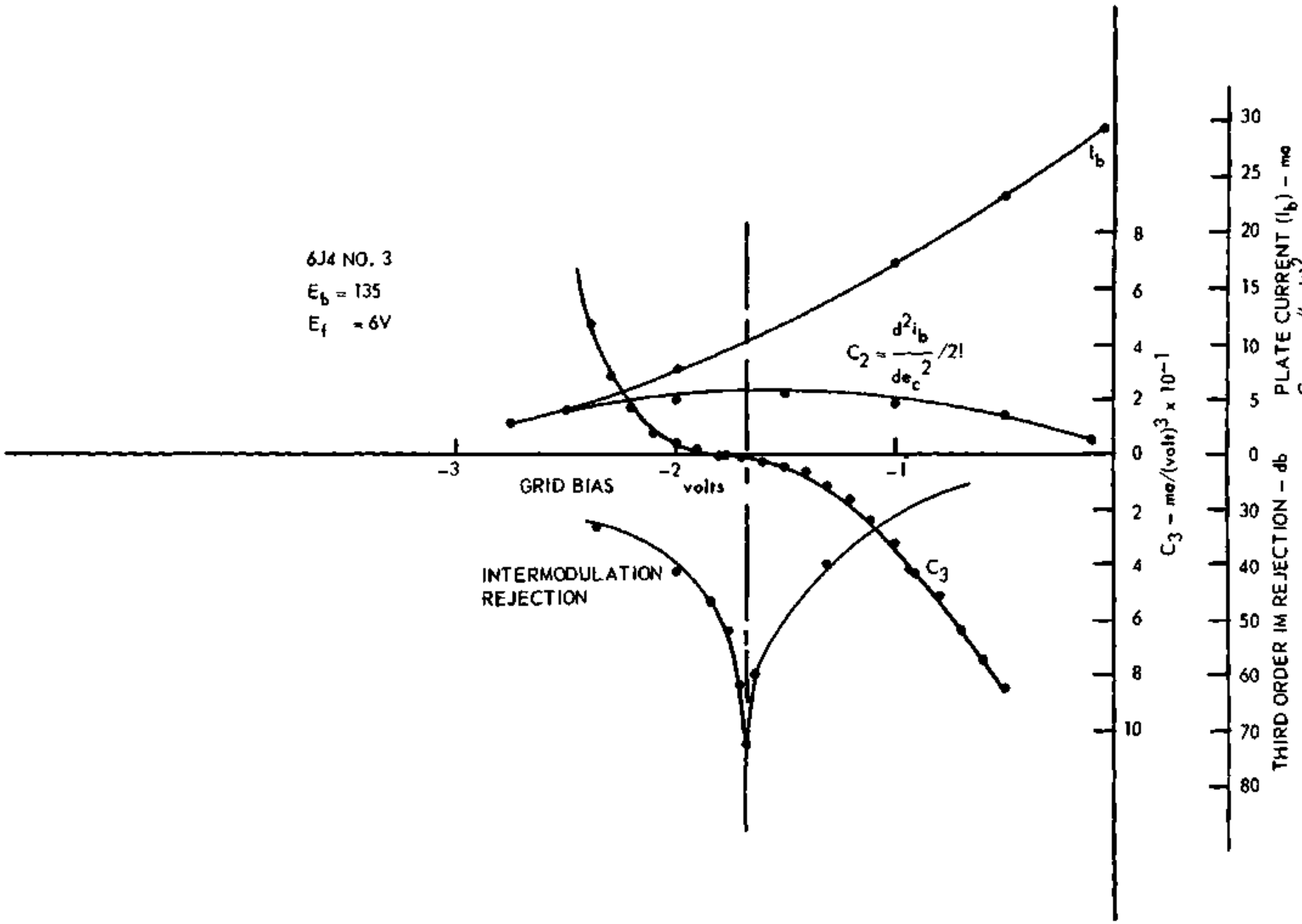

Figure16. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6J4.

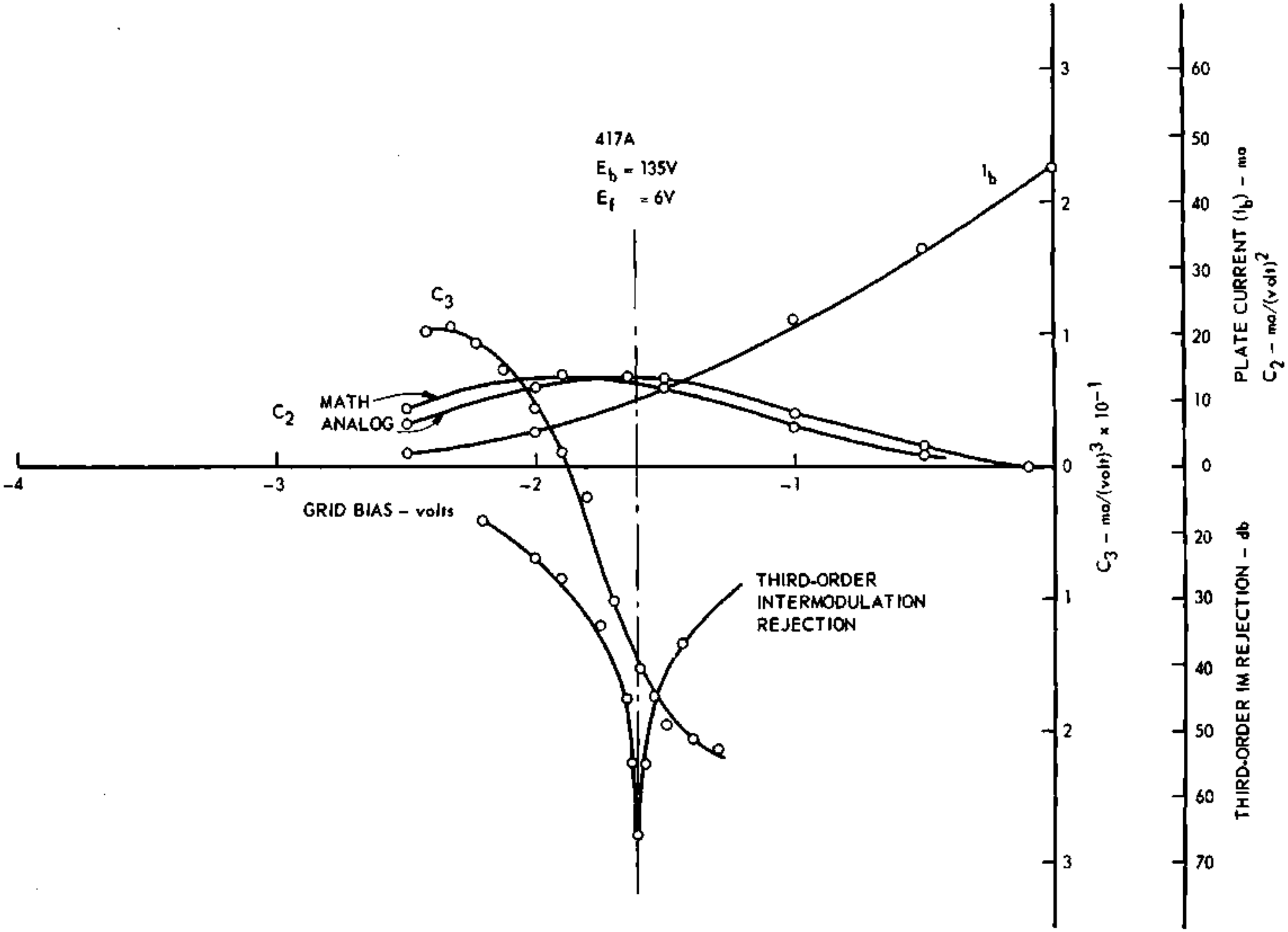

Figure 17. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 417A.

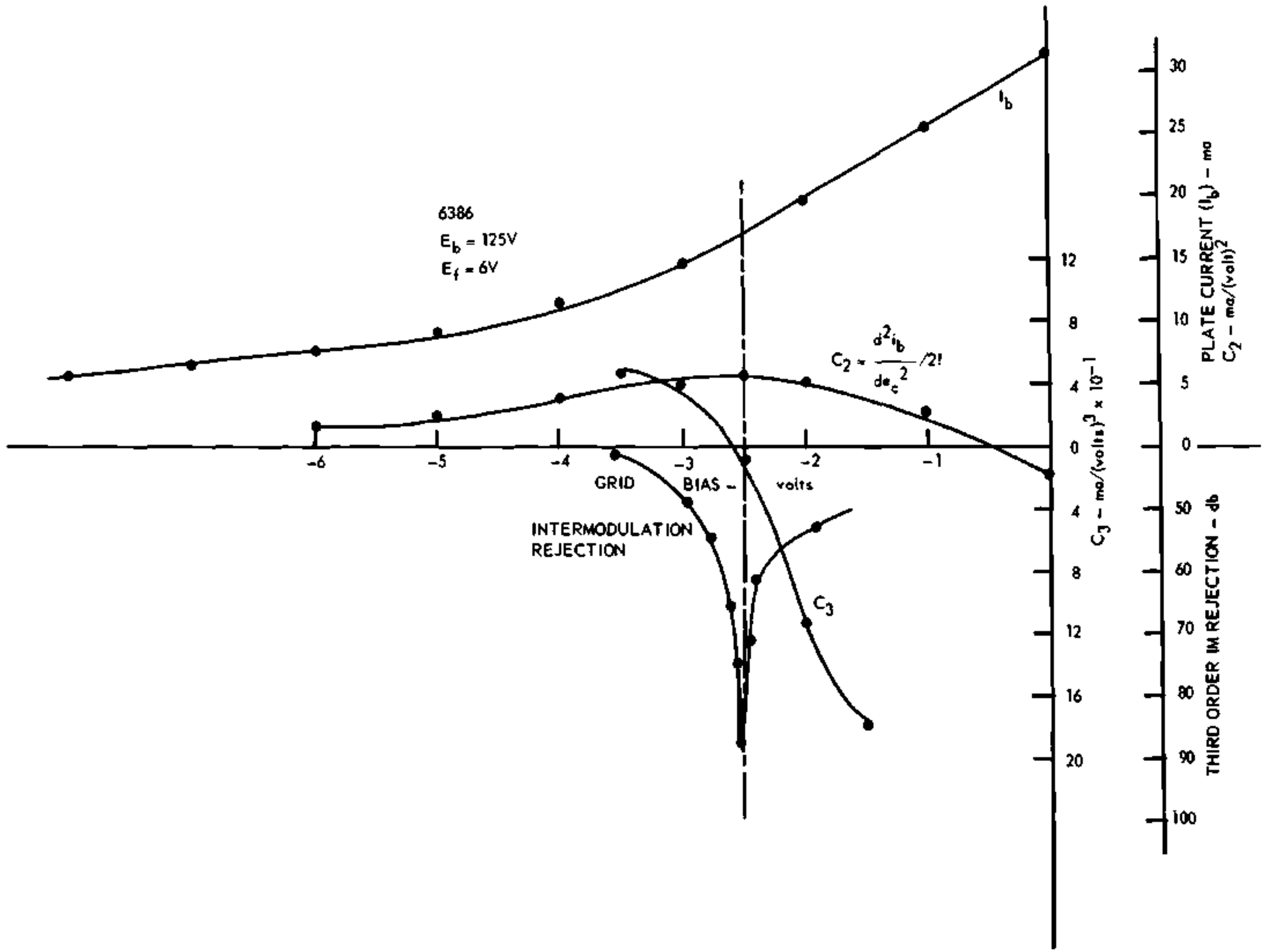

Figure 18. Composite Characteristics of Distortion Coefficients and Intermodulation Rejection for 6386.

Table 6

GRAMM-TSHEBYSHEFF WORKSHEET

Step 1. Divide the interval of the independent variable into 14 equal parts and find the corresponding values of the dependent variable. These values are listed as $\phi(X)$. The transformed interval is $0 \leq X \leq 1$.

Step 2. $a_1 = \frac{1}{D_1} \Sigma N_1(X)\ \phi(X)$ $a_2 = \frac{\Sigma N_2(X)\phi(X)}{D_2}$

X	$N_0(X)$	$N_1(X)$	$N_2(X)$	$N_3(X)$	$N_4(X)$	$N_5(X)$	$N_6(X)$	$N_7(X)$	$\phi(X)$
0	1	7	91	637	1001	11011	1859	13	
1/14	1	6	52	91	-429	-12584	-3718	-39	
2/14	1	5	19	-245	-869	-10769	- 715	17	
3/14	1	4	- 8	-406	-704	- 484	2288	31	
4/14	1	3	-29	-427	-249	8261	2561	3	
5/14	1	2	-44	-343	251	11000	650	-25	
6/14	1	1	-53	-189	621	7425	-1625	-25	
7/14	1	0	-56	0	756	0	-2600	0	
8/14	1	-1	-53	189	621	- 7425	-1625	25	
9/14	1	-2	-44	343	251	-11000	650	25	
10/14	1	-3	-29	427	-249	- 8261	2561	- 3	
11/14	1	-4	- 8	406	-704	484	2288	-31	
12/14	1	-5	19	245	-869	10769	- 715	-17	
13/14	1	-6	52	- 91	-429	12584	-3718	39	
1	1	-7	91	-637	1001	-11011	1859	-13	
D_i	15	40	408	3060	6460	116280	38760	646	

Table 6 (Continued)

GRAM-TSHEBYSHEFF WORKSHEET

Step 3. $\phi^*(X) = b_0 + b_1 X + \ldots + b_7 X^7$

$$b_2 = \frac{41580\ a_2 + 242550\ a_3 + \ldots + 25912964\ a_7}{6435}$$

j	M_0	M_1	M_2	M_3	M_4	M_5	M_6	M_7		
1	1								a_0	
2	1	- 4290							a_1	
3	1	- 13860	41580						a_2	
4	1	- 31240	2 42550	- 1 61700					a_3	
5	1	- 62125	9 06675	- 14 40600	1 44060				a_4	
6	1	-1 19881	28 55475	- 79 54170	18 15156	- 36 30312			a_5	
7	1	-2 39071	85 15661	- 363 03120	139 83424	- 621 18672	18 82384		a_6	
8	1	-5 21191	259 12964	-1554 40054	885 56083	-6481 78762	428 24236	-9 41192	a_7	
	1	2145	6435	6435	1287	6435	585	45		
b_j										

Table 7. Digital Computer Program for the Evaluation of Vacuum Tube Nonlinearities

The following program computes from transfer curve data: (1) a seventh degree Tshebysheff-Polynomial approximation to the transfer curve using the "Gram-Tshebysheff Worksheet", (2) the approximating polynomial at fifty values of bias, (3) residues at the fifteen input data points, and (4) $C_N(C_1$ or C_2 or $C_3)$ at forty bias values. This program is written in the Bell General Purpose System Language for the IBM 650.

ORDERS	OUTLINE OF PROGRAM
1 - 5	Start the Program and call in new data.
6 - 39	Evaluation of $a_i = \frac{1}{D_i} \sum N_i(x)\, \emptyset(x)$ of step two of polynomial worksheet.
40 - 61	Evaluation b's from worksheet and print out as coefficient of polynomial. Print out of input data.
62 - 82	Evaluation of Tshebysheff Polynomial at 50 points by formulas $\left\{ \{ [\![\, [\![\, [\, (xb_7 + b_6) + b_5] x + b_4]\!] \, x + b_3]\!] \, x + b_2 \} \, x + b_1 \right\} x + b_o$
83 - 94	Calculation of residues between Tshebysheff Polynomial and experimental points.
95 - 126	Calculation of Espley method for C_1, C_2 or C_3 at 40 different bias points.

STORAGE

A. Storage of constants for Gram-Tshebysheff Polynomial fitting routine from 15-point worksheet.

Table 7 (Continued)

STEP 2

1.	Constants	of row	one	are	stored	from	200 to 207.	
2.	"	" "	two	"	"	"	210 to 217.	
.							.	.
.							.	.
.							.	.
17.	"	" "	17	"	"	"	390 to 397.	

STEP 3

1.	Constants	of row	one	are	stored	from	400 to 407.	
2.	"	" "	two	"	"	"	410 to 417.	
.							.	.
.							.	.
.							.	.

9. Constants of row nine are stored from 480 to 487.

B. Storage Experimental Data

1. Fifteen points of transfer curve from 520 to 534.
2. Fifteen - 1 = Fourteen in 190.
3. Bias for first point on transfer curve in 191.
4. Increment of bias between data in 192.
5. Desired evaluation increment of Tshebysheff Polynomial in 193.
6. Special program constant ($.0001 \times 10^{-50}$) in 194.
7. 195 data for run identification.

C. Espley Evaluation Constants

1. 570 . . . . . . - 820
2. 571 . . . . . . + 10088
3. 572 . . . . . . - 58428
4. 573 . . . . . . + 209712
5. 574 . . . . . . - 280392
6. 575 . . . . . . + .0013778659 (for Δe_g = .1 volt)

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
1.	+0 800 049 000	Program point No. 49	
2.	+7 000 190 195	Read in data from 190 to 195, stop if data is not available	14, initial bias, bias increment, bias evaluation increment, program constant
3.	+7 000 520 534	Read in data from 520 to 534	Points from transfer curve
4.	+7 000 570 575	Read in data from 570 to 575	Espley constants
5.	+7 300 190 195	Punch out from 190 to 195	
6.	+9 800 001 000	Program Point No. 1	
7.	+9 100 100 000	Modify the next non nine order of form X $\overset{A}{XXX}$ $\overset{B}{XXX}$ $\overset{C}{XXX}$ by adding one to A part	
8.	+2 200 520 000	Multiply number in 200 by number in 520 and store in 000	$N_o(0)\ \phi(0)$
9.	+9 100 100 000	Add one to A part of next non nine order	
10.	+4 210 521 000	Multiply No. in 210 by No. in 521 and add previous result store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14)$
11.	+9 100 100 000	Add one to A part of next non nine order	
12.	+4 220 522 000	Multiply No. in 220 by No. in 522 and add previous results store in 000	$N_o(9)\ \phi(0) + N_o(1/14)\ \phi(1/14) + N_o(2/14)\ \phi(2/14)$
13.	+9 100 100 000	Add one to A part of next non nine order	
14.	+4 230 523 000	Multiply number in 230 by number in 523 and add previous result and store 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \ldots\ldots N_o(3/14)\ \phi(3/14)$
15.	+9 100 100 000	Add one to A part of next non nine order	
16.	+4 240 524 000	Multiply number in 240 by number in 524 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \ldots\ldots N_o(4/14)\ \phi(4/14)$

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
17.	+9 100 100 000	Add one to A part of next non nine order	
18.	+4 250 525 000	Multiply number in 250 by number in 525 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(5/14)\ \phi(5/14)$
19.	+9 100 100 000	Add one to A part of next non nine order	
20.	+4 260 526 000	Multiply number in 260 by number in 526 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(6/14)\ \phi(6/14)$
21.	+9 100 100 000	Add one to A part of the next non nine order	
22.	+4 270 527 000	Multiply number 270 by number in 527 add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(7/14)\ \phi(7/14)$
23.	+9 100 100 000	Add one to A part of the next non nine order	
24.	+4 280 528 000	Multiply number 280 by number in 528 add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(8/14)\ \phi(8/14)$
25.	+9 100 100 000	Add one to A part of the next non nine order	
26.	+4 290 529 000	Multiply number in 290 by number in 529 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(9/14)\ \phi(9/14)$
27.	+9 100 100 000	Add one to A part of the next non nine order	
28.	+4 300 530 000	Multiply number in 300 by number in 530 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(10/14)\ \phi(10/14)$

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
29.	+9 100 100 000	Add one to A part of next non nine order	
30.	+4 310 531 000	Multiply number in 310 by number in 531 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(11/14)\ \phi(11/14)$
31.	+9 100 100 000	Add one to A part of the next non nine order	
32.	+4 320 532 000	Multiply number in 320 by number in 532 and add previous results and store 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(12/14)\ \phi(12/14)$
33.	+9 100 100 000	Add one to A part of the next non nine order	
34.	+4 330 523 000	Multiply number in 330 by number in 533 and add previous result and store in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(13/14)\ \phi(13/14)$
35.	+9 100 100 000	Add one to A part of next non nine order	
36.	+4 340 534 000	Multiply number in 340 by number in 534 and add previous result in 000	$N_o(0)\ \phi(0) + N_o(1/14)\ \phi(1/14) + \dots + N_o(1)\ \phi(1)$
37.	+9 100 011 000	Add to B & C parts of the next non nine order	
38.	+3 000 390 500	Divide number in 000 by number in 390 and store in 500	$a_i = \frac{1}{D_i} \sum N_i(x)\ \phi(x)$ in 50 i
39.	+8 101 008 001	Transfer to PP 1 and repeat loop 8 times	9 100 100 000 orders interchange N_1's for N_o's, N_2's for N_1's, ... N_7's for N_6's
40.	+9 800 002 000	Program Point 2	

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
42.	+2 400 500 000	Multiply number in 400 by number in 500 and store 000	$M_o(1)\ a_o$
43.	+9 200 100 000	Add 1 to the A part of the next non nine order	
44.	+4 410 501 000	Multiply number in 410 by number in 501 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1$
45.	+9 200 100 000	Add 1 to the A part of the next non nine order	
46.	+4 420 502 000	Multiply number in 420 by number in 502 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(3)\ a_2$
47.	+9 200 100 000	Add 1 to the A part of the next non nine order	
48.	+4 430 503 000	Multiply number in 430 by number in 503 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(4)\ a_3$
49.	+9 200 100 000	Add 1 to the A part of the next non nine order	
50.	+4 440 504 000	Multiply number in 440 by number in 504 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(5)\ a_4$
51.	+9 200 100 000	Add 1 to the A part of the next non nine order	
52.	+4 450 505 000	Multiply number in 450 by number in 505 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(6)\ a_5$
53.	+9 200 100 000	Add 1 to the A part of the next non nine order	

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Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
54.	+4 460 506 000	Multiply number in 460 by number in 506 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(7)\ a_6$
55.	+9 200 100 000	Add one to the A part of the next non nine order	
56.	+4 470 507 000	Multiply number in 470 by number in 507 and add previous result and store in 000	$M_o(1)\ a_o + M_o(2)\ a_1 + \ldots + M_o(8)\ a_{(o)}$
57.	+9 200 011 000	Add one to the B & C part of the next non nine order	
58.	+3 000 480 510	Divide number in 300 by number in 480 and store in 510	$\frac{1}{M_9} \sum_{N=0}^{8} M^{(N)} a_n$
59.	+8 201 008 002	Transfer to program point 2 and repeat loop 8 times	Transfer to program point 2 eight times to calculate b_o, b_1, b_2, . . . b_7 by changing M_o to M_1, M_1 to M_2 . . . M_6 to M_7
60.	+7 300 510 518	Punch out numbers in 510 to 518	
61.	+7 300 520 534	Punch out numbers in 520 to 534	
62.	+7 201 510 500	Move number in 500 to 510	Store b_o which is value of polynomial at 0 in 500
63.	+7 215 520 150	Move number in 520 and next fourteen numbers to 150 and next fourteen addresses	
64.	+7 208 510 170	Move number in 510 and next seven numbers to 170 and next seven addresses	

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
65.	+3 192 193 182	Divide number in 192 by number in 193 and store in 182	Data increment $\div$ evaluation increment
66.	+2 182 190 180	Multiply number in 182 by number in 190 and store 180	$\left(\frac{\text{data increment}}{\text{evaluation increment}}\right)$ (14)
67.	+7 300 180 180	Punch out number in 180	
68.	+3 901 180 180	Divide number in 901 by number in 180 and store in 180	$\frac{\text{evaluation increment}}{\text{data increment (14)}}$
69.	+1 900 180 181	Add number in 900 to number in 180 and store in 181	Store 180 also in 181 since 900 has 0
70.	+2 182 194 182	Multiply number in 182 by number in 194 and store in 182	$\frac{\text{data increment}}{\text{evaluation increment}}(.0001)10^{-50}$
71.	+9 800 020 000	Program Point 20	
72.	+1 180 900 000	Add number in 180 to number in 900 and store in zero	Store 180 in 000 since number in 900 = 0
73.	+5 177 176 000	Multiply number in 177 by previous result and add number in 176 and store in 000	$b_7x + b_6$
74.	+5 180 175 000	Multiply number in 180 by previous result and add number in 175 and store in 000	$(b_7x + b_6)x + b_5$
75.	+5 180 174 000	Multiply number in 180 by previous result and add number in 174 and store in 000	$[(b_7x + b_6)x + b_5]x + b_4$
76.	+5 180 173 000	Multiply number in 180 by previous result and add number in 173 and store in 000	$[[(b_7x + b_6)x + b_5]x + b_4]x + b_3$
77.	+5 180 172 000	Multiply number in 180 by previous result and add number in 172 and store in 000	$[[[(b_7x + b_6)x + b_5]x + b_4]x + b_3]x + b_2$

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
78.	+5 180 171 000	Multiply number in 180 by previous result and add number in 171 and store in 000	$\{[[[(b_7x + b_6)x + b_5]x + b_4]x + b_3]x + b_2x\} + b_1$
79.	+9 100 001 000	Add one to the C part of the next non nine order	Change address of evaluation storage from 501 to 500
80.	+5 180 170 501	Multiply number in 180 by previous result and add number in 170 and store in 501	$(\{[[[(b_7x + b_6)x + b_5]x + b_4]x + b_3]x + b_2\}x + b_1)x + b_o$
81.	+1 180 181 180	Add number in 180 to number in 181 and store in 180	Change x to 2x to 3x . . . 49x
82.	+8 101 049 020	Transfer to PP 20 and repeat loop 49 times	Calculate evaluation 50 times
83.	+9 800 021 000	P.P. 21	
84.	+9 100 011 000	Add 1 to B and C parts of the next non nine order	
85.	+9 200 100 000	Add 1 to A part of the next non nine order	
86.	-1 500 150 150	Subtract number in 150 from number in 500 and store in 150	To form first residue subtract data value from value of polynomial at first bias point
87.	+8 101 015 022	Transfer to program part 22 and repeat loop 15 times	
88.	+8 000 000 023	Transfer to Program Point 23	

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
90.	+0 200 182 000	Add number in 182 to next order	Add $\frac{\text{data increment}}{\text{evaluation increment}}$ (.0001) 10^{-50} to next order
91.	+8 200 019 021	Transfer the Program Part 21 and repeat loop 19 times	Order 90 causes order 91 to become 820 x 019 021 where x = $\frac{\text{data increment}}{\text{evaluation increment}}$. The x increment places the right evaluation in order 86 to form the residues by using order 85 to modify 86.
92.	+9 800 023 000	Program Point 23	
93.	+9 600 002 000	Reset index register 2	
94.	+7 300 150 164	Punch out numbers in 150 to 164	Punch out residues
95.	+7 300 500 504	Punch out numbers in 500 to 504	Punch out first five evaluations of polynomial
96.	+3 905 902 649	Divide number in 905 by number in 902 and store in 649	10/2 = 5
97.	+2 649 193 649	Multiply number in 649 by number in 193 and store in 649	Multiply 5 times evaluation increment 193
98.	+1 649 191 649	Add number in 649 to number in 191, store in 649	Add initial bias (191) to 5 times evaluation increment
99.	+9 800 031 000	Program Point 31	
100.	+9 200 110 000	Add one to A and B parts of the next non nine order	
101.	-1 500 510 556	Subtract number in 510 from number in 500, store in 556	$I_{11} - I_1$

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
102.	+5 570 900 559	Multiply number in 570 by previous results and add number in 900 and store 559	$-820(I_{11} - I_1)$
103.	+9 200 110 000	Add one to A and B parts of the next non nine order	
104.	-1 501 509 558	Subtract number in 509 from number in 501 and store in 558	$I_{10} - I_2$
105.	+5 571 900 559	Multiply 571 by previous result and add number in 900 and store in 559	$10088(I_{10} - I_2)$
106.	+9 200 110 000	Add one to A and B parts of the next non nine order	
107.	-1 502 508 560	Subtract number in 508 from number in 502 and store in 560	$I_9 - I_3$
108.	+5 572 900 561	Multiply 572 by previous result and add number in 900 and store in 561	$-58428(I_9 - I_3)$
109.	+9 200 110 000	Add one to A and B parts of the next non nine order	
110.	-1 503 507 562	Subtract number in 507 from number in 503 and store in 562	$I_8 - I_4$
111.	+5 573 900 563	Multiply number in 573 by previous result and add number in 900 and store in 563	$209712(I_8 - I_4)$
112.	+9 200 110 000	Add one to A and B parts of next non nine order	
113.	-1 504 506 564	Subtract number in 506 from number in 504 and store in 564	$I_7 - I_5$
114.	+5 574 900 565	Multiply number in 574 by previous result and add number in 900 and store 565	$-280392(I_7 - I_5)$
115.	+1 557 559 000	Add number in 579 to number in 559 and store in 000	$-820(I_{11} - I_1) + 10088(I_{10} - I_2)$

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
116.	+1 561 000 000	Add number in 561 to number in 000 and store in 000	$-820(I_{11} - I_1) + 10088(I_{10} - I_2)$ $- 58428(I_9 - I_3)$
117.	+1 563 000 000	Add number in 563 to number in 000 and store in 000	$-820(I_{11} - I_1) + 10088(I_{10} - I_2)$ $- 58428(I_9 - I_3)$ $+ 209712(I_8 - I_4)$
118.	+1 565 000 000	Add number in 565 to number in 000 and store in 000	$-820(I_{11} - I_1) + 10088(I_{10} - I_2)$ $- 58428(I_9 - I_3)$ $+ 209712(I_8 - I_4)$ $- 280392(I_7 - I_5)$
119.	+2 000 575 651	Multiply number in 000 by number in 575 and store in 651	Multiply previous result by 575 which equals $\frac{1}{725760(\Delta E_g)^3}$ (note that this value in 575 must be given to the machine and will vary with evaluation increment)
120.	+9 200 010 000	Add one to B part of the next non nine order	
121.	+7 201 505 650	Move number in 505 to address 650	Move value of I_6 to 650
122.	+7 300 649 651	Punch out numbers in 649 to 651	Punch out bias, current, C_3
123.	+1 649 193 649	Add number in 649 to number in 193 and store in 649	Add bias evaluation increment to last bias for evaluation of C_3
124.	+8 201 040 031	Transfer to Program point 31 and repeat loop 40 times	Find C_3 for 40 values of bias

Table 7 (Continued)

MASTER PROGRAM FOR VACUUM TUBE CHARACTERISTIC EVALUATION -- IBM 650 COMPUTER

Order No.	Order	Operation	Result of Operation
125.	+7 300 545 549	Punch out numbers from 545 to 549	Punch out last five current evaluations from Tshebysheff Polynomial
126.	+8 000 000 049	Transfer to Program Point 49	Return to program point 49 and program will continue if more data is available at readin unit.